Classical and quantum cosmologies with dynamical fundamental constants

dr **Adam Balcerzak**

Self-Report

Information on professional achievements

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1 Personal Data

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2 Education and Degrees

Ph.D. in Physical Sciences	Faculty of Mathematics and Physics, University of Szczecin, 2009, Ph.D. thesis title: *"Higher-order brane gravity theories"*, Supervisor: prof. dr hab. Mariusz P. Dąbrowski.
Master Degree	Faculty of Physics, Astronomy and Informatics, Nicolaus Copernicus University in Toruń, 2005, Master Thesis title: *"Geometric structures in quantum mechanics"*, Supervisor: prof. dr hab. Dariusz Chruściński.

3 Professional Experience

assistantship 01.10.2005 - 30.09.2010	Institute of Physics, Faculty of Mathematics and Physics, University of Szczecin.
adjunct from 01.10.2010	Institute of Physics, Faculty of Mathematics and Physics, University of Szczecin.

4 Scientific achievement being the basis for the habilitation procedure

The scientific achievement, in accordance with the art. 16 paragraph 2 of the Act of March 14th, 2003, concerning the scientific degrees and titles (Dz. U. no. 65, item 595, as amended), is the series of publications entitled:

Classical and quantum cosmologies with dynamical fundamental constants

In the following list I report the data regarding the publications entering the habilitation procedure, together with the description of my personal contribution to each of them. For each publication, I provide the Impact Factor by year of publication derived from Journal Citation Report (JCR).

SA1. A. Balcerzak, M. P. Dąbrowski, "Redshift drift in varying speed of light cosmology", Physics Letters B728, 15-18 (2014).
https://doi.org/10.1016/j.physletb.2013.11.029

In this paper we investigate the impact of assumed varying speed of light on the redshift drift relation. We show that for the decreasing with cosmological expansion speed of light the effective pressure of the matter that fills the universe decreases as well. On the other hand the growing of speed of light results in an increase in the matter effective pressure. My contribution consisted in: performing most of the calculations, discussing the results obtained and giving active contribution to writing the article.

My percentage contribution is estimated at about 80%.
Impact Factor: 5.81 (2014 JCR).

SA2. A. Balcerzak, M.P. Dąbrowski, "A statefinder luminosity distance formula in varying speed of light cosmology", Journal of Cosmology and Astroparticle Physics (JCAP) 06, 035 (2014).
https://doi.org/10.1088/1475-7516/2014/06/035

In this paper we express the luminosity distance formula in terms of the statefinders jerk, snap and lerk in the theory with varying speed of light. We show how such formula can be used to test the influence of varying speed of light on cosmological evolution. In particular we prove that the effect of varying speed of light can be isolated with the values of the up to second order expansion parameters. My contribution consisted in: performing part of the calculations, discussing the obtained results and giving active contribution to writing the article.

My percentage contribution is estimated at about 60%.
Impact Factor: 5.81 (2014 JCR).

SA3. A. Balcerzak, M. P. Dąbrowski, V. Salzano, "Modelling spatial variations of the speed of light", Annalen der Physik 2017, 529, 1600409.
https://doi.org/10.1002/andp.201600409

In this paper we investigate spatial and temporal variation of the speed of light in the framework of inhomogeneous Stephani model and check if such model can in principle be falsified with cosmological observations. My contribution consisted in: performing part of the numerical and analytical calculations, discussing the obtained results and giving active contribution to writing the article.

My percentage contribution is estimated at about 33%.
Impact Factor: 2.557 (2017 JCR).

SA4. K. Leszczyńska, M.P. Dąbrowski, A. Balcerzak, "Varying constants quantum cosmology", Journal of Cosmology and Astroparticle Physics (JCAP) 02, 012 (2015).
https://doi.org/10.1088/1475-7516/2015/02/012

In this paper we investigate quantum cosmology based on Wheeler-DeWitt equation in theories with varying speed of light and the dynamical gravitational constant. We show that in the considered model the minisuperspace potentials are of the tunneling type and calculate the probability of the tunneling of the universe from a singular state (with vanishing scale factor) to a state described with Friedmann geometry for scenarios assuming varying speed of light and varying gravitational constant. In particular we show that the probability is higher for the models with increasing value of the speed of light while it is strongly suppressed in scenarios assuming diminishing speed of light compared to the standard cosmological models with constant fundamental parameters. In the scenarios with dynamical gravitational constant the tunneling probability behaves in exactly opposite way. My contribution consisted in: performing part of the calculations and discussing the methods and the results obtained.

My percentage contribution is estimated at about 30%.
Impact Factor: 5.634 (2015 JCR).

SA5. A. Balcerzak, "Non-minimally coupled varying constants quantum cosmologies", Journal of Cosmology and Astroparticle Physics (JCAP) 04, 019, (2015).
https://doi.org/10.1088/1475-7516/2015/04/019

In this paper we discuss near Big Bang singularity quantum cosmology based on Wheeler-DeWitt equation in models with varying speed of light and the dynamical gravitational constant with both parameters being represented by non-minimally coupled scalar fields. The classical evolution contains both the pre-big-bang contraction and the post-big-bang expansion phases separated by the curvature singularity. We show that in the framework of the models with dynamical fundamental constants the universe can in principle pass form the pre-big-bang contraction to post-big-bang expansion in the process of the "over the singularity" quantum scattering. My contribution consisted in: performing all of the calculations, discussing the obtained results and writing the article.

My percentage contribution is 100%.
Impact Factor: 5.634 (2015 JCR).

SA6. K. Marosek, M.P. Dąbrowski, A. Balcerzak, "Cyclic multiverses", Monthly Notices of the Royal Astronomical Society, 461, 2777-2788 (2016).
https://doi.org/10.1093/mnras/stw1424

In this paper we consider cyclic universes in the context of theories with dynamical fundamental constants. We show that in such models a singularity in the matter density parameter can be regularised even in the presence of the strong curvature singularity. Next, we extend the considered model to a multiverse case and investigate the behaviour a doubleverse (a multiverse consisting of two universes) with exactly the same geometrical evolution of its components while the evolution of the dynamical fundamental constants is different in both universes. An important achievement of this work is presenting a model where the drop of the entropy in one of the universe is counterbalanced by the increase of the entropy in the other one. As a result, the total entropy of the considered doubleverse remains constant and the evolution of the doubleverse respects the second law of thermodynamics. My contribution consisted in: performing part of the calculations and discussing the results obtained.

My percentage contribution is estimated at about 33%.
Impact Factor: 4.961 (2016 JCR).

SA7. S. Robles-Perez, A. Balcerzak, M.P. Dąbrowski, M. Kraemer, "Interuniversal entanglement in a cyclic multiverse", Physical Review D95, 083505 (2017).
https://doi.org/10.1103/PhysRevD.95.083505

In this paper we consider a multiverse consisting of the two identical non-interacting and quantum mechanically entangled universes evolving in exactly the same cyclic way. By using the tools to investigate the thermodynamic of the entanglement we calculate the temperature and the entropy of entanglement. We show that the entropy of entanglement reaches its maximum value for Big Bang and Big Crunch singularity as well as for the time of maximal expansion of both of these universes. The entropy of entanglement for Big Rip singularity vanishes while the temperature of entanglement reaches infinity for each of the considered classical singularities as well as for the time of maximal expansion. This suggests that the entropy may serve as a better estimator of "quantumness" than the temperature.

My percentage contribution is estimated at about 20%.
Impact Factor: 4.394 (2017 JCR).

4.1 Description of the academic achievement

4.1.1 Introduction - the dynamical fundamental constants

Formulating the theory with varying speed of light and the dynamical gravitational constant faces numerous conceptual problems since it necessary leads to violation of the foundations of *all* the contemporary physical theories. It is perfectly obvious that understanding the whole impact on the structure of physical theories of the assumption that the fundamental constants may be represented by some dynamical parameters goes far beyond the scope of a single research project. The very fact that each theoretical description resting on such a speculative assumption developed within a single research project would never comprise all the important aspects of the physical reality can not be a reason to give up research on the theories of such type. In this work I will present the results of my investigation of the problems of cosmology based on the theory with varying speed of light and the dynamical gravitational constant $[SA1 - SA7]$.

Theories with dynamical speed of light c Theories with varying speed of light (VSL) were proposed as an alternatives to a standard inflation explaining of the horizon and the flatness problem.

In the paper [1] and interesting mechanism was proposed in which the sudden increase in the speed of light is a consequence of the local spontaneous Lorentz symmetry breaking which occurs in a very early universe and is triggered by the first-order phase transition for some critical moment in time t_c. The large value of the speed of light causally connects all the regions of the universe which solves the horizon and the flatness problem. After the critical moment t_c is reached the local Lorentz symmetry is restored and the speed of light attains its standard value.

Another example of the theory with varying speed of light is a theory proposed in the paper [2, 3]. A starting point for this theory is a choice of the light frame (a preferred frame usually identified with the cosmological frame) and then the replacement of the constant speed of light c in the action with a space-time dependent function. The form of the action for such a VSL model is:

$$S = \int dx^4 \Big(\sqrt{-g} \Big(\frac{\psi(R + 2\Lambda)}{16\pi G} + \mathcal{L}_M \Big) + \mathcal{L}_\psi \Big) , \tag{1}$$

where $\psi(x^\mu) = c^4$. Here, the dynamical variables are: the metric components $g_{\mu\nu}$, the matter fields contained in the matter part of the lagrangian $\mathcal{L}_M$ and the function ψ. The Riemman tensor is computed with respect to the metric $g_{\mu\nu}$ with assumption of constant ψ. Such a way of computing of the Riemann tensor components is allowed only in the light frame. With the assumption that

$\mathcal{L}_\psi$ does not explicitly contain the metric components $g_{\mu\nu}$ the variation of the action (1) gives the following field equations:

$$G_{\mu\nu} - g_{\mu\nu}\Lambda = \frac{8\pi G}{\psi} T_{\mu\nu}. \tag{2}$$

Thus, the abovementioned variational procedure leads to the standard in a from Einstein equations with the constant speed of light replaced by the space-time dependent function $c^4(x^\mu) = \psi(x^\mu)$. The gravity theory governed by the field equations (2) provides solution of the horizon problem, the flatness problem and the classical cosmological constant problem.

Another example of the gravity theory with varying speed of light is the bimetric gravity theory proposed in [4]. It defines the two metrics linked with each other by the following formula:

$$\hat{g}_{\mu\nu} = g_{\mu\nu} + \beta\psi_\mu\psi_\nu, \tag{3}$$

where ψ_μ is a vector field and $\beta > 0$ is a constant. The gravitational field is described by the metric $g_{\mu\nu}$ while the matter fields respect the geometry described by $\hat{g}_{\mu\nu}$. The action that defines such a bimetric theory is:

$$S_{\text{tot}} = S_{\text{gr}}[g] + S_\psi[\psi, g] + S_{\text{matter}}[\hat{g}, \text{matter fields}]. \tag{4}$$

The term $S_{\text{gr}}[g]$ is a standard Einstein-Hilbert action and is calculated with respect to the metric $g_{\mu\nu}$. The term $S_\psi[\psi, g]$ is the Maxwell-Proca action that defines the dynamics of the vector field ψ_μ. The term $S_{\text{matter}}[\hat{g}, \text{matter fields}]$ is a matter field action and depends on the metric $\hat{g}_{\mu\nu}$. In the considered theory the conservation laws are: $\hat{\nabla}_\nu T^{\mu\nu}_{\text{matter}}[\hat{g}] = 0$, where $\hat{\nabla}_\nu$ denotes the covariant derivative compatible with the metric $\hat{g}_{\mu\nu}$. This implies that the trajectory of the test particle follows geodesics of the geometry described by $\hat{g}_{\mu\nu}$. In the frame defined by the metric $g_{\mu\nu}$ the standard Einstein equations are fulfilled with the matter sources which violate the causality since the the maximal speed of the matter resulting from the metric $\hat{g}_{\mu\nu}$ is higher than the speed of gravitational waves determined by the metric $g_{\mu\nu}$. On the other hand in the frame defined by the metric $\hat{g}_{\mu\nu}$ the matter does not violate causality. The cosmological scenarios based on the gravity theory given by (4) contains a phase of inflation (which occurs when the field ψ_μ is non vanishing) sufficient for solving the horizon problem, the flatness problem and the magnetic monopole problem. The inflation comes to an end when both causal structures defined by metrics $g_{\mu\nu}$ and $\hat{g}_{\mu\nu}$ become identical and this occurs for $\psi_\mu = 0$.

An interesting generalization of the bimetric theories is the theory given in [5] with the metric depending on the energies E of the particles used by the freely falling observer to determine the geometry of spacetime. Such theory assumes the modified equivalence principle which states that the local physics in the frame of freely falling observer respects the "flat" geometry of the deformed special relativity given by the following metric:

$$ds^2 = -\frac{(dx^0)^2}{f^2(E/E_{PL})} + \frac{(dx^i)^2}{g^2(E/E_{PL})}, \tag{5}$$

where f and g are some general functions of energy E and E_{PL} jest the parameter of the theory (the same for all inertial observers). Moreover, the deformed special relativity theory assumes an equivalence of the inertial frames and the correspondence principle which states that for $E/E_{Pl} \rightarrow 0$ the deformed special relativity transforms into standard special relativity. In consequence the dispersion relation takes the following modified form:

$$E^2 f^2(E/E_{Pl}) - p^2 g^2(E/E_{Pl}) = m^2\ . \tag{6}$$

The cosmological scenarios in such theories are based on the modified Friedmann metric. The comoving horizon for such a metric is given by:

$$r_h = \frac{c(E)H^{-1}}{a/g}, \tag{7}$$

where a is the scale factor and $c(E) = g/f$ is the speed of light that depends on E. The choice of a sufficiently quickly decreasing in the early universe function $g(E)$ allows for solving the horizon problem.

Theories with dynamical gravitational constant G The problem of modeling of the dynamics of the gravitational constant G can be addressed in the framework of scalar-tensor gravity theories [6, 7, 8]. Due to their simple structure such theories are often used for modeling phenomena whose explanation in the framework of Einstein General Relativity theory requires making some speculative assumptions. Scalar-tensor theories naturally arise in the context of the theories with higher number of dimensions such like Kaluza-Klein or string theory (as a result of dimensional reduction). The action describing the scalar-tensor theory is given by:

$$S = \frac{1}{16\pi}\int d^4x\sqrt{-g}\left[f\left(\phi\right)R - \frac{\omega\left(\phi\right)}{\phi}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - 2\Lambda\left(\phi\right) + L_m(\psi, g_{\mu\nu})\right], \tag{8}$$

where ϕ is the scalar field, f, ω and Λ are some arbitrary functions of the scalar field, and L_m is the matter lagrangian. For $f \to \phi$, $\omega = const.$ and for $\Lambda \to 0$ the action (8) reduces to form of the well-known Brans-Dicke action:

$$S = \int d^4x \frac{1}{16\pi}\sqrt{-g}\left(\phi R - \frac{\omega}{\phi}\partial_\mu\phi\partial^\mu\phi + L_m\right). \tag{9}$$

In Brans-Dicke theory the gravitational interaction is mediated by the metric tensor $g_{\mu\nu}$ which determines the geodesics, and the scalar field ϕ whose inverse gives the value of the gravitational constant G. Such theory is a special case of the Horndeski theory which constitutes the most general four-dimensional scalar-tensor theory governed by the second-order field equations [9].

4.1.2 Dark energy in the context of theories with varying speed of light

The analysis of the data taken form the observations of Supernovae type Ia provides strong evidence for accelerating expansion of the Universe [10, 11]. In order to explain this fact in the framework of the homogeneous and isotropic cosmology based on standard General Relativity theory one has to assume an existence of the dark energy matter component uniformly filling the space. The analysis of the data taken form the observations of temperature anisotropies in the CMB confirms the existence of dark energy [12], however, the constraint on the amount of dark energy coming form the CMB data is much weaker than the analogous constraint resulting from the Supernovae type Ia data. Another observations based on measurement of baryonic acoustic oscillations (BAO) [13, 14] provides an independent test which also seems to confirm the existence of the dark energy component. The cosmological tests based on the observations of the large scale structure of the Universe also favor cosmological models with dark energy [15].

The redshift drift cosmological tests is based on measurements of rate of change of redshift of distant cosmological objects. The time dependence of redshift of distant cosmological objects was first postulated by Sandage [16]. The possible cosmological consequences of this effect was investigated by Loeb [17]. An idea to analyse the data taken form two light cones separated by some time interval leads to a concept of an independent testing of cosmological models. The redshift drift measurement is planned as part of the CODEX experiment (Cosmical Dynamics Experiment) [18, 19]. The goal of the experiment is to measure the absorption spectra of quasars (Lyman-α forest [20]) every few years and then (basing on the cross-correlation calculated for the measured spectra) to estimate the redshift drift of the observed quasars for the assumed time interval.

In paper [$SA1$] we consider the redshift drift test within the model with varying speed of light defined in papers [2, 3] described with Friedmann metric:

$$ds^2 = -(dx^0)^2 + a^2(t)\left[\frac{dr^2}{1-Kr^2} + r^2(d\theta^2 + \sin^2\theta d\varphi^2)\right] \ , \tag{10}$$

where $a(t)$ is the scale factor, $K = \pm 1$ is the curvature index and $dx^0 = c(t)dt$. In the considered model the equations of motion are the standard Friedmann equations with the time dependent speed of light $c = c(t)$, given by:

$$\varrho(t) = \frac{3}{8\pi G}\left(\frac{\dot{a}^2}{a^2} + \frac{Kc^2(t)}{a^2}\right) \ , \tag{11}$$

$$p(t) = -\frac{c^2(t)}{8\pi G}\left(2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{Kc^2(t)}{a^2}\right) \ , \tag{12}$$

Same as in [2, 3] we assume that the speed of light is a function of the scale factor a:

$$c = c_0\left(\frac{a}{a_0}\right)^n \ , \tag{13}$$

where $a_0 = a(0)$ is the current value of the scale factor and $c_0 = c(0)$ is the current value of the speed of light. We notice that in the limit $n \to 0$ the equations of motion (11) and (12) transforms into standard Friedmann equations with constant speed of light. For the flat universe ($K = 0$) filled with dust and dark energy in the form of cosmological constant the eq. (11) can be rewritten in the following form:

$$H^2(z) = H_0^2\left[\Omega_{m0}(1+z)^3 + \Omega_{\Lambda_0}\right] \ , \tag{14}$$

where $H = \frac{\dot{a}}{a}$ is the Hubble parameter, H_0 is the current value of the Hubble parameter, $\Omega_{m0} = \frac{8\pi G}{3H_0^2}\varrho_{m0}$ and $\Omega_{\Lambda_0} = \frac{\Lambda_0 c_0^2 a_0^{2n}}{3H_0^2}$ are the current values of the dimensionless density parameters for the dust matter (with density ρ_0) and the cosmological constant Λ_0. In paper [$SA1$] we show that the dependence of the redshift drift on the redshift is given by the following formula:

$$\frac{\Delta z}{\Delta t_0} = \frac{\Delta z}{\Delta t_0}(z,n) = H_0(1+z) - H_0\sqrt{\sum_{i=1}^{i=k}\Omega_{wi}(1+z)^{3(w_{eff}+1)}} \ , \tag{15}$$

where $\Omega_{w1} = \Omega_{m0}$, $\Omega_{w2} = \Omega_{\Lambda_0}$ and the effective barotropic index w_{eff} is:

$$w_{eff} = w_i + \frac{2}{3}n \ , \tag{16}$$

with $w_1 = 0$ for the dust matter and $w_2 = -1$ for the dark energy in the form of cosmological constant Λ_0.

The expression (15) shows that in models with decreasing speed of light the dust effectively behaves like matter with slightly negative pressure while the dark energy represented by the cosmological constant effectively behaves like phantom matter [21]. On the other hand in models with increasing speed of light the pressure of any kind of matter effectively grows. We also show (Fig. 1) that in the context of the CODEX experiment models with varying speed of light characterized with the parameter $|n| < 0.045$ are indistinguishable form the model with the cosmological constant and the dust matter which assumes constant speed of light (the ΛCDM model). For higher value of n parameter (the case with growing speed of light $c(t)$) the drift in the considered model become more like the drift in the model with constant speed of light and the dust matter only (the CDM model).

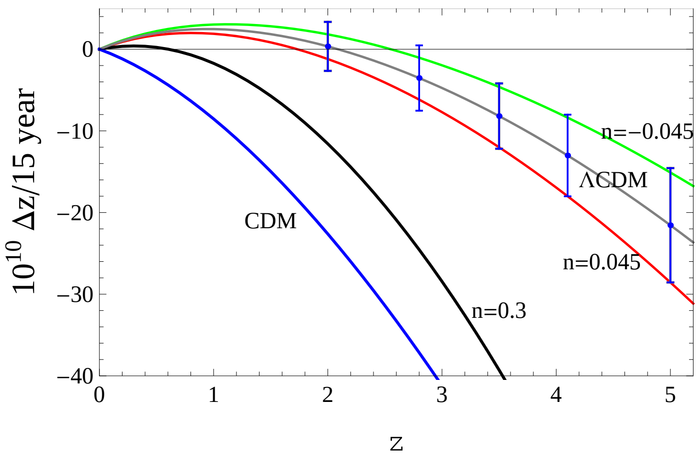

Figure 1: The drift (formula (15)) for 15 years time interval for different value of n. The negative value of n corresponds to $\dot{c} < 0$. We used the error bars estimated in paper [22] for the model with the cosmological constant Λ_0, the dust matter and the constant speed of light (ΛCDM).

The quantity that characterizes the distances between the cosmological objects is the luminosity distance. The value of the luminosity distance can be observationally determined for supernovae type Ia due to the known value of their absolute magnitude [10]. On the other hand the comparison of the luminosity distance calculated for any cosmological model under examination with the luminosity distances observed for supernovae type Ia provides constraints on parameters of the examined model. In order to fit the Friedmann cosmological model to the supernovae type Ia data one has to assume an existence of the uniformly distributed dark energy matter component which is a source of the accelerating expansion of the Universe [11].

In paper $[SA2]$ we examine the influence of the varying c on the interpretation of the observed luminosity distances for supernovae type Ia in the framework of the models proposed in papers [2, 3] with the spacetime metric given by (10) and the speed of light c given by (13). We assume the considered model to contain the dust matter, the cosmological constant and the nonvanishing curvature term ($K \neq 0$). We show that in such a model the luminosity distance

$D_L(z)$ is the following function of the statefinders jerk, snap and lerk:

$$D_L(z) = c_0 \frac{z}{H_0} \times \Big\{ 1 + \frac{1}{2}(1 - q_0 - n)z$$
$$+ \frac{1}{6}\Big[q_0(3q_0 + 2n + 1) + (n^2 - j_0 - 1) - \Omega_{K0}\Big] z^2$$
$$+\frac{1}{24}\Big[5j_0(2q_0 + 1) - s_0 - 15q_0^2(q_0 + 1) + 2(1 - q_0)$$
$$+2\Omega_{K0}(3q_0 + 3n + 1) + n(3j_0 - 9q_0^2 - 7q_0 + 1)$$
$$-n^2(3q_0 + n + 2)\Big] z^3 + \frac{1}{120}\Big[-6 - l_0 + \Omega_{K0}^2$$
$$-\Omega_{K0}(5 - 10j_0 + 30n + 25n^2 + 40q_0 + 50nq_0 + 45q_0^2)$$
$$+s_0(11 + 4n + 15q_0) - j_0(27 - 10j_0 + 27n + 6n^2)$$
$$+5j_0q_0(22 - 8n - 21q_0) + n(-5 + 5n + 5n^2 + n^3)$$
$$+nq_0(29 + 21n + 4n^2 + 81q_0 + 18nq_0 + 60q_0^2)$$
$$+q_0(6 + 81q_0 + 165q_0^2 + 105q_0^3)\Big] z^4 + O(z^5)\Big\} \quad . \tag{17}$$

where $\Omega_{K0} = \frac{Kc_0^2}{H_0^2 a_0^2}$ is the curvature density parametes, H_0 is the current value of the Hubble parameter $H = \frac{\dot{a}}{a}$, q_0 is the current value of the deceleration parameter $q = -\frac{1}{H^2}\frac{\ddot{a}}{a} = -\frac{\ddot{a}a}{\dot{a}^2}$, j_0 is the current value of the jerk parameter $j = \frac{1}{H^3}\frac{\dddot{a}}{a} = \frac{\dddot{a}a^2}{\dot{a}^3}$, s_0 is the current value of the snap parameter $s = -\frac{1}{H^4}\frac{\ddddot{a}}{a} = -\frac{\ddddot{a}a^3}{\dot{a}^4}$ and l_0 is the current value of the lerk parameter $l = \frac{1}{H^5}\frac{a^{(5)}}{a} = \frac{a^{(5)}a^4}{\dot{a}^5}$.

The statefinders were originally introduced as a tool to effectively distinguish between the dynamical dark energy models and the models in which the dark energy is represented by cosmological constant [23]. The form of the second-order therm in the expansion (17) indicates that the expansion acceleration rate may by overestimated if the actual time dependence of the speed of light with $n < 0$ was neglected (the case with decreasing c). On the other hand neglecting the time dependence of c for $n > 0$ (the case with increasing c) may lead to underestimating the acceleration rate of the expansion. The effect of the varying c on the estimation of the values of the higher-order expansion parameters is much more complex and involves higher-order terms in n (n^2, n^3 i n^4). The relations between the higher-order expansion parameters and the physical parameters Ω_{m0}, $\Omega_{\Lambda 0}$ and Ω_{K0} are given by the following formulas:

$$\Omega_{m0} - \Omega_{K0} + \Omega_{\Lambda 0} = 1 \quad , \tag{18}$$

$$\Omega_{\Lambda 0}(1 + n) = \frac{1}{2}\Omega_{m0} - q_0 + n\Omega_{K0} \quad , \tag{19}$$

$$\Omega_{K0} = \frac{3}{2}\Omega_{m0} - (q_0 + n) - 1 \quad , \tag{20}$$

$$j_0 = \Omega_{m0} + \Omega_{\Lambda 0}(n + 1) - n\Omega_{K0} \quad , \tag{21}$$

$$s_0 = 3\Omega_{m0} + n\Omega_{K0} - (n + 1)\Omega_{\Lambda 0} + q_0 j_0 \quad , \tag{22}$$

$$s_0 = \frac{5}{2}\Omega_{m0} + q_0(j_0 + 1) = 4\Omega_{m0} + j_0(q_0 - 1). \tag{23}$$

Given the values of the parameters Ω_{m0} and $\Omega_{\Lambda 0}$ allow (with the help of formula (18)) to determine the parameter Ω_{K0}. Comparison of (19) with the second-order term of the expansion (17) allows to calculate the parameters q_0 and n. Given the parameter Ω_{m0} only the value of n can be isolated by using relation (21) provided we know the value of the third-order therm in expansion (17). If the fourth-order therm in expansion (17) is measured application of (21) and (23) allows again to isolate the value of n.

All the models with varying c considered in the literature so far allowed only temporal dependence of the speed of light. In papers [24, 25] there was presented a method to estimate the value of the speed of light with the barionic acoustic oscillations (BAO) data in the cosmological model described by Friedmann metric. Including the spatial dependence of the speed of light requires modifying the Albrecht-Magueijo model to comply with an inhomogeneous spatial geometry. A motivation for considering such a models comes from the observational data which may indicate an existence of the large-scale structure anisotropies which manifest through the spatial dipol dependence of the fine structure constant α (the distribution of α distinguishes a certain direction in space) [26, 27, 28, 29, 30, 31] and through the large-scale flow of the dark matter [32, 33] and the dark energy [34, 35].

In paper [$SA3$] we use as a cosmological model the inhomogeneous and spherically symmetric, conformally flat Stephani metric [36] which complies with an assumption of the spatial dependence of the speed of light c. The Stephani metric is given by:

$$ds^2 = -c_0^2 \frac{a^2}{\dot{a}^2} \left[\frac{\left(\frac{V}{a}\right)^{\cdot}}{\left(\frac{V}{a}\right)} \right]^2 dt^2 + \frac{a^2}{V^2} \left[dr^2 + r^2 d\Omega^2 \right], \tag{24}$$

where

$$V(t,r) = 1 + \frac{1}{4} k(t) r^2 , \tag{25}$$

and $(\ldots)^{\cdot} \equiv \partial/\partial t$. The function $a(t)$ plays the role of the generalized scale factor while $k(t)$ is the time dependent curvature index, r is the radial coordinate and c_0 is the actual value of the speed of light. The metric (24) fulfil the Einstein equations with matter in the form of perfect fluid. The cosmological equations for the metric (24) generalize the standard Friedmann equations to the form with the homogeneous energy density $\varrho(t)$ and the inhomogeneous pressure $p(t,r)$ that depends on time and the distance to the centre of symmetry:

$$\varrho(t) = \frac{3}{8\pi G} \left[\frac{\dot{a}^2(t)}{a^2(t)} + \frac{k(t) c_0^2}{a^2(t)} \right], \tag{26}$$

$$\begin{aligned} p(t,r) &= w_{eff}(t,r) \varrho(t) c_0^2 \\ &\equiv \left[-1 + \frac{1}{3} \frac{\dot{\varrho}(t)}{\varrho(t)} \frac{\left[\frac{V(t,r)}{a(t)}\right]}{\left[\frac{V(t,r)}{a(t)}\right]^{\cdot}} \right] \varrho(t) c_0^2, \end{aligned} \tag{27}$$

where we define the effective inhomogeneous barotropic index $w_{eff}(t,r)$. For quantitative considerations we took a subclass of the Stephani models with $k(t) = \beta a(t)$. In this case the metric (24) reduces to [37, 38, 39]:

$$ds^2 = -\frac{c_0^2}{V^2} dt^2 + \frac{a^2(t)}{V^2} \left(dr^2 + r^2 d\Omega^2 \right). \tag{28}$$

The special case of the Stephani model given by metric (28) was fitted to the observational data in papers [38, 40, 41, 42]

We consider three different dependencies of c on space-time coordinates: the constant speed of light, time dependence of the speed of light $c = c_0 a^n(t)$ and space-time dependence of the speed of light $c = c_0/V(t,r)$. We generalize the scheme introduced in paper [24, 25] which uses the relation between the maximal value of the angular diameter distance D_A and the speed of light given by:

$$c(t) = D_A H \ , \tag{29}$$

to the case of an inhomogeneous Stephani model (model IIA) given by (28). The relation between the maximal value of the angular diameter distance D_A as a function of redshift and the speed of light for the model given by (28) is:

$$c(t, r) = D_A(HV - \dot{V}). \tag{30}$$

Based on the formulas above we define a quantity Δ_c:

$$\Delta_c = \frac{D_A H}{c_0} \ , \tag{31}$$

that characterizes the deviation of the investigated model form the homogeneous and isotropic model (based on Friedmann metric) with constant c. It should be stressed that the nonvanishing value of Δ_c indicates either an existence of inhomogeneity or variability of c or both these effects at the same time. By fitting the model to the observational data (supernovae type Ia, BAO, CMB shift parameter) we estimate the values of the parameters of the model and then (with the help of the values of those parameters) we calculate the redshift z_M at which the angular diameter distance D_A reaches its maximum, and the quantity Δ_c for the three mentioned above space-time dependencies of the speed of light c. As it was shown in [25], the future radio telescope network SKA (Square Kilometre Array) will be able to detect one-percent deviation from the homogeneous model with constant c at 3σ confidence level. The calculated values of Δ_c give deviatios on the level of about 10%. This means that the models investigated in paper [$SA3$] can in principle be falsified by the observational data collected by SKA. If no signal of such order of magnitude will be detected, it will be a clear signature that the postulated type of inhomogeneity does not exist or that the value of the inhomogeneity is very small. However, this will not rule out the possibility of the time dependence of the speed of light. The problem of the dipole anisotropy of the fine structure constant α will also remain unresolved.

4.1.3 Quantum universe and multiverse in the context of theories with varying speed of light and dynamical cosmological constant

Quantum cosmogenesis in theories with dynamical fundamental constants. In paper [$SA4$] we quantize the theory with varying speed of light and the dynamical gravitational constant described in papers [2, 3]. In order to perform the quantisation procedure we formulate our theory in terms of the lagrangian formalism. The action of the considered model contains: the Einstein-Hilbert term, the cosmological constant term, the matter term and the Gibbons-Hawking boundary term with appropriate modification which takes into account the fact that the speed of light and the gravitation constant may vary:

$$S = \int_M d^4x\sqrt{-g}R\frac{c^3}{16\pi G} - \int_M \rho c\sqrt{-g}d^4x + \int_{\partial M} d^3x\sqrt{h}K\frac{c^3}{8\pi G} \ , \tag{32}$$

where R is the Ricci scalar, ρ is the matter density and K is the trace of the extrinsic curvature. In the action (32) the speed of light c and the gravitational constant G may generally be dependent on space and time coordinates: $c = c(x^\mu)$ oraz $G = G(x^\mu)$. Since we are interested in cosmological scenarios we assume that the spacetime is described by the Friedmann metric of the form:

$$ds^2 = -(ct)^2 + a^2(t)[d\chi^2 + S^2(\chi)d\Omega^2] \ , \tag{33}$$

where

$$S(\chi) = \begin{cases} \sin\chi, & \text{k=+1}, \\ \chi, & \text{k= 0}, \\ \mathrm{sh}\chi, & \text{k= -1} \end{cases} \tag{34}$$

In order to quantize the model we use the standard canonical quantization procedure which results in writing an appropriate Wheeler-DeWitt equation [43]. Since the variability of fundamental constants may by related with the expansion of the universe we assume that the variability of c and G can be modeled with the following functions: $c = c_0 a^n$ and $G = G_0 a^q$. Consequently the configurational space of our model is given by the one-dimensional minisuperspace and the Wheeler-DeWitt equation is formally equivalent to the Schrödinger equation describing the particle moving in one-dimensional potential $U(a)$ of tunneling type:

$$\left[\hbar^2 \frac{\partial^2}{\partial a^2} - U(a)\right] \Psi(a) = 0, \tag{35}$$

where

$$U(a) = -\left(\frac{3V_3 c^2(a) a}{4\pi G(a)}\right)^2 \left[k c^2(a) - \frac{\Lambda}{3} a^2 c^2(a) - \frac{8\pi G(a)}{3} \varrho(a) a^2\right]. \tag{36}$$

The dependence of the matter density ρ on the scale factor a was obtained by integrating the continuity equation which for the assumed variability of c and G is given by:

$$\dot{\varrho} + 3\frac{\dot{a}}{a}\left(\varrho + \frac{p}{c^2(t)}\right) + \varrho\frac{\dot{G}(t)}{G(t)} = \frac{(3k - \Lambda a^2)}{4\pi G(t) a^2} c(t)\dot{c}(t). \tag{37}$$

The resulting potential $U(a)$ is:

$$\begin{aligned} U(a) = {} & -K_0^2 a^{2(3n+1-q)} \\ & \left(\frac{3w+1}{2n+3w+1} k - \frac{\Lambda(w+1)}{2n+3(w+1)} a^2 - \frac{8\pi G_0}{3c_0^2} \frac{C}{a^{3w+1+2n}}\right), \end{aligned} \tag{38}$$

where

$$K_0 = \frac{3V_3 c_0^3}{4\pi G_0}. \tag{39}$$

By application of WKB method we calculate the probability of tunneling of the universe characterized by vanishing value of the scale factor $a = 0$ to the state described by the Friedmann geometry with a particular value of the scale factor:

$$\begin{aligned} P & \simeq \exp\left[-\frac{2}{\hbar}\int_0^{a_t} \sqrt{2(E - U(a))} da\right] \\ & = \exp\left[-\frac{2K_0}{\hbar}\int_0^{a_t} a^{3n+1-q}\left(\frac{\Lambda(w+1)}{2n+3(w+1)} a^2 - \frac{3w+1}{2n+3w+1} k\right)^{1/2} da\right]. \end{aligned} \tag{40}$$

The expression (40) shows that the mentioned probability of tunneling is higher for the models with c growing as the universe expands ($n > 0$) while attains much smaller values for the models with c diminishing during the expansion ($n < 0$) (Fig. 4). On the other hand the probability

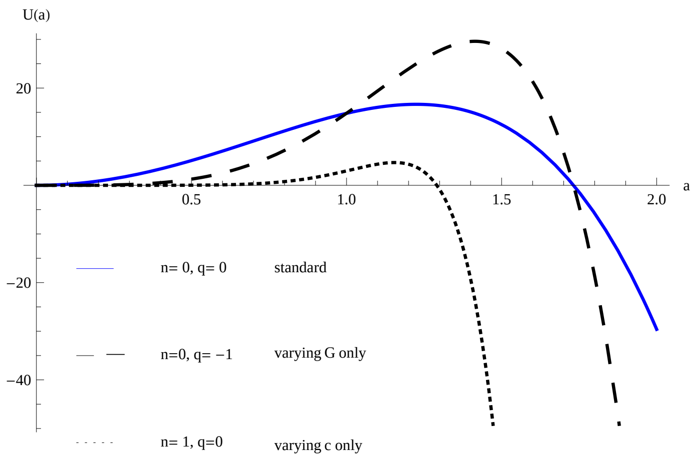

Figure 2: Potential $U(a)$ of tunneling type for dust universe ($w = 0$) for three different sets of model parameters: blue line represents the universe with constant G and c; dashed lines represent universes with only one of the fundamental parameters c or G varying ($C = 0$).

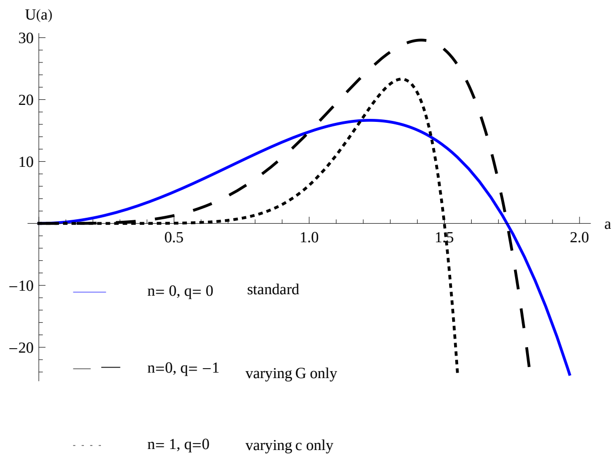

Figure 3: Potential $U(a)$ of the tunneling type for the universe filled with radiation ($w = 1/3$) for three different sets of the model parameters: blue line represents the universe with constant G and c; dashed lines represent universes with only one of the fundamental parameters c or G varying ($C = 0$).

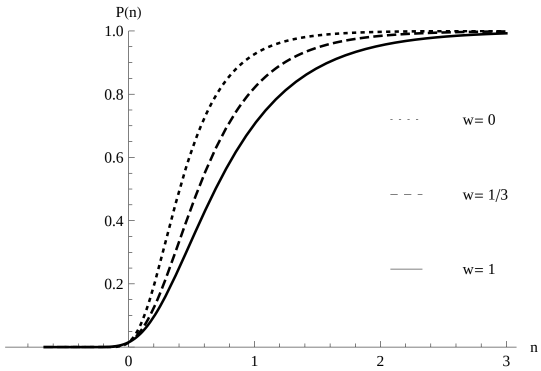

Figure 4: Tunneling probability (40) for the model with varying c ($k = +1$, $q = 0$, $\Lambda = 5$, $\hbar = G_0 = c_0 = 1$) for three different cases: the universe filled with dust ($w = 0$), radiation ($w = 1/3$) and stiff fluid ($w = 1$). In the scenario with diminishing c ($n < 0$) the tunneling probability is much lower than the tunneling probability in the scenario with c growing ($n > 0$).

of tunneling is higher for the models with G diminishing as the universe expands ($q < 0$) while attains smaller value for the models with G growing during the expansion ($q > 0$) (Fig. 5). An exception is the universe filled with stiff fluid ($w = 1$) in which the tunneling probability vanishes for sufficiently large as well as for sufficiently small value of q.

A qualitatively different scenario of quantum cosmogenesis naturally arise in theories in which the speed of light c and the gravitational constant G are represented by scalar fields [$SA5$]. The way in which c and G enter the action of the considered theory results in non-minimally coupled gravity model:

$$S = \int \sqrt{-g} \left(\frac{e^{\phi}}{e^{\psi}} \right) [R + \Lambda + \omega(\partial_\mu \phi \partial^\mu \phi + \partial_\mu \psi \partial^\mu \psi)] \, d^4x \ , \tag{41}$$

where ω is a parameter of the theory and the relations between $c(x^\mu)$ and $G(x^\mu)$ and the scalar fields $\phi(x^\mu)$, $\psi(x^\mu)$ are given by: $c^3 = e^{\phi}$ oraz $G = e^{\psi}$. As it was shown in [$SA5$] the theory defined by the action (41) is equivalent to the Brans-Dicke theory (the eq. (2.3) in [$SA5$]). In the context of such a model we examined the classical and quantum evolution of the flat universe described by Friedmann metric. The form of the hamiltonian (formula (2.9) in [$SA5$]) indicates that in the considered model the expansion of the universe in the high-curvature regime (near the big bang singularity) formally corresponds to a scattering of a particle on one-dimensional exponential potential barrier in the minisuperspace. The cosmological evolution includes the phase of the pre-big-bang contraction and the phase of post-big-bang expansion with both phases separated by the curvature singularity (Fig. 6). In the moment of the curvature singularity the speed of light c attains infinite value while the dynamical gravitational constant G vanishes. This means that the transition from the pre-big-bang contraction to the post-big-bang expansion occurs within a newtonian regime. In the considered model the curvature singularity appears for $I \to \infty$ (in our model the particle moves along the direction parameterised by variable I) while the low-curvature regime (far form the curvature singularity) is represented by the region given by $I \to -\infty$. With both regimes there are associated the following asymptotic values of the momentum π_I:

$$\pi_I = \begin{cases} \pi_I^\infty & \text{contraction before big-bang} \\ -\pi_I^\infty & \text{expansion after big-bang} \end{cases}$$

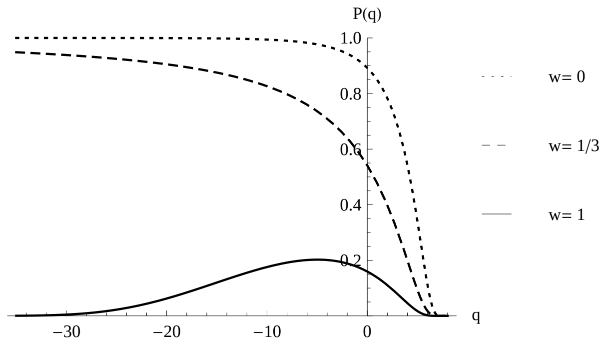

Figure 5: Tunneling probability (40) for the model with varying c and varying G ($k = +1$, $n = 2$, $\Lambda = 2$, and $\hbar = G_0 = c_0 = 1$) for three different cases: the universe filled with dust ($w = 0$), radiation ($w = 1/3$) and stiff fluid ($w = 1$). In the scenario with diminishing G ($q < 0$) the tunneling probability is much higher than the tunneling probability in the scenario with G growing ($q > 0$). For the universe filled with stiff fluid the tunneling probability vanishes for sufficiently large value of $|q|$.

for the high-curvature regime and:

$$\pi_I = \begin{cases} \pi_I^{\infty} e^{-I} & \text{contraction before big-bang} \\ -\pi_I^{\infty} e^{-I} & \text{expansion after big-bang} \end{cases}$$

for the low-curvature regime where π^{∞} is a positive parameter of the model. The procedure of canonical quantisation of the investigated theory leads to the Wheeler-DeWitt equation which formally describe the statrionary scattering of a particle on the exponential potential barrier:

$$\left\{ \frac{1}{4} \left[\frac{1}{m} \left(\frac{\partial^2}{\partial I^2} - \frac{\partial^2}{\partial J^2} \right) + \frac{\partial^2}{\partial B^2} \right] - \bar{\Lambda} e^{-2I} \right\} \Phi = 0 \tag{42}$$

where I, J and B are coordinates on the minisuperspace. From the all possible solution of (42) we pick the solution Φ which asymptotically for $I \to \infty$ fulfil the eigenproblem of the momentum operator $\hat{\pi}_I$:

$$\hat{\pi}_I \Phi = \pi_I^{\infty} \Phi \,. \tag{43}$$

(asymptotically Φ is an eigenfunction of $\hat{\pi}_I$ to the eigenvalue π_I^{∞}) and this way it represent the collapsing universe in the high-curvature regime. On the other hand the solution Φ in the low-curvature regime is given by the superposition of the state Ψ_2 which describe the particle approaching the potential barrier and state Ψ_1 which represent the particle scattered and reflected by the barrier. Such interpretation immediately results for the following properties of the states Ψ_1 and Ψ_2:

$$\Phi = \Psi_1 + \Psi_2 \,, \tag{44}$$

where Ψ_1 and Ψ_2 are solution of the eigenproblem of the momentum operator $\hat{\pi}_I$:

$$\hat{\pi}_I \Psi_1 = -\pi_I^{\infty} e^{-I} \Psi_1 \tag{45a}$$

$$\hat{\pi}_I \Psi_2 = \pi_I^{\infty} e^{-I} \Psi_2 \tag{45b}$$

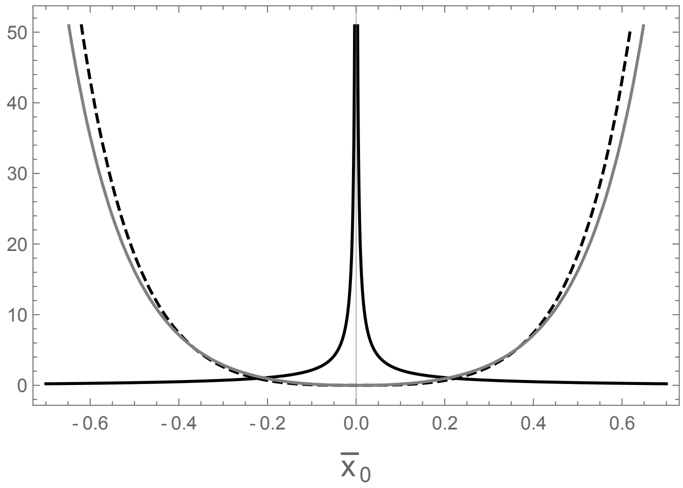

Figure 6: Scale factor a (grey), speed of light c (continues line) and gravitational constant G (dashed line) before ($\bar{x}^0 < 0$) and after ($\bar{x}^0 > 0$) the big-bang singularity.

(here Ψ_1 is an eigenfunction of the momentum operator $\hat{\pi}_I$ to the eigenvalue $-\pi_I^\infty e^{-I}$ while Ψ_2 is an eigenfunction of the same operator for the eigenvalue $\pi_I^\infty e^{-I}$). Ψ_2 represents the universe collapsing onto the curvature singularity while Ψ_1 represent the expanding universe in the evolution phase occurring after the curvature singularity. We also calculate the reflection coefficient which gives the probability of the transition from the contraction phase (a phase which occurs before the curvature singularity) to the expansion phase (a phase which occurs after the curvature singularity):

$$R = \frac{|\Psi_1|^2}{|\Psi_2|^2} = e^{-2\pi\pi_I^\infty} . \tag{46}$$

Thus, we show that the quantized gravity theories with varying c and G include a scenario in which the universe passes over the curvature singularity from the contraction phase to the expansion phase in the process of the quantum scattering on the exponential potential barrier in the minisuperspace. An analogous behavior appears in the case of the ekpyrotic [46, 47] and cyclic scenarios [44, 45] where in the high curvature regime the universe, after having undergone the phase of the accelerated contraction, faces the big-crunch/bang singularity and then bounces to enter the standard expanding phase. On the other hand, we encounter a qualitatively different behavior in the pre-big bang scenario within the framework of string cosmology where the pre-big bang phase corresponds to expanding and accelerating universe approaching the strong coupling regime [48].

Regularization of singularities and cyclic universes in models with dynamical c and G. We consider the cyclic models [$SA6$] in the framework of the cosmological model with varying c and G defined in [2, 3] which include periodically appearing strong singularities. In the case of the sinusoidal model which evolution is specified by the ansatz

$$a(t) = a_0 \left| \sin\left(\pi \frac{t}{t_c}\right) \right| , \tag{47}$$

the cyclically emerging singularity is the big bang/big crunch. For the tangential model described by the ansatz

$$a(t) = a_0 \left| \tan \left(\pi \frac{t}{t_s} \right) \right| \tag{48}$$

the singularities that appear cyclically is the big bang and the big rip (here a_0 is a constant). We show that assuming varying G and imposing that G reaches infinity in the moment when the strong singularity occurs may lead to a peculiar regularization of the singularities which appear in our scenario. The dependence of G on the scale factor a that leads to the mentioned regularization is for the sinusoidal model given by:

$$G\left(t\right) = \frac{G_0}{a^2(t)} \ , \tag{49}$$

while for the tangential model is:

$$G\left(t\right) = \frac{4G_s}{\sin^2\left(2\pi \frac{t}{t_s}\right)} \ , \tag{50}$$

As a result of this regularization at the moment when the scale factor reaches zero, the energy density and pressure remain finite (equations (4.10) and (4.11) or (4.30) and (4.31) in [$SA6$]). Additionally, in the sinusoidal model the null, weak and strong energy conditions are fulfilled (equations (4.6) and (4.7) in [$SA6$]). In the case of the tangential model the null and strong energy conditions are satisfied in the big bang singularity while they are broken in the big rip (equations (4.32) - (4.35) in [$SA6$]). A similar scenario arises in the ekpyrotic and the cyclic models [44, 45, 46, 47] which appear in the context of the effective string cosmologies. In such theories the choice of the appropriate form of coupling of the scalar field to matter leads to a situation in which the density of dust matter and radiation reaches a finite value at the moment of the singularity which appears for the vanishing scale factor.

We examined the scenario of the multiverse [$SA6$] in the context of the theory with varying c and G defined in [2, 3]. The universes which make up the investigated multiverse evolve in such a way that the total entropy of the multiverse that is the sum of the entropies of individual universes remains constant in the course of the evolution. Thus, the considered system does not violate the second law of thermodynamics.

Thermodynamical considerations based on the principle of energy conservation (the first law of thermodynamics) and the Albrecht and Magueijo model lead to compact expression for the entropy of individual universes. For varying c model the entropy is proportional to the natural logarithm of the term linear in c (equation (5.13) in [$SA6$]):

$$S(t) = \frac{2}{w} \frac{pV}{T} \ln c(t) = \frac{2}{w} N k_B \ln c(t) \ \ , \tag{51}$$

where N is the number of particles, w is the barotropic index of the matter that fills the universe and k_B is the Boltzmann constant. In the model that allows variability of G the entropy is proportional to the natural logarithm of the inverse of G (equation (5.17) in [$SA6$]):

$$S(t) = N k_B \ln \left[\frac{1}{G(t)} \right] \ . \tag{52}$$

We consider a multiverse consisting of the two universes in the theory with the varying c. Since the entropy of a single universe is a function of c only the entropy can grow or decrease depending on the assumed scenario of variability of c. According to the second law of thermodynamics the total entropy can not decrease and therefore from the whole set of possible scenarios of varying c the only scenarios that survive are those that do not violate the second law of thermodynamics. Admitting the existence of other universes within the multiverse allows to reformulate this condition. We postulate the total entropy of the multiverse to be constant. This allows to take into account scenarios in which the decrease of the entropy in one universe is compensated by the increase of the entropy in the other one so that the total entropy remains constant during the evolution. The solution that respects the above assumption includes the cyclically evolving speed of light in both universes which make up the multiverse (equations (5.21), (5.22) and (5.26),(5.27) in [$SA6$]) and the cyclically evolving scale factor a (equation (5.35)). Consequently the entropies in individual universes evolve in a cyclic manner (equation (5.28) and (5.29)). In the case of the multiverse considered in the context of varying G the solutions that respect the condition of constant total entropy include scenarios with singular G: while in one of the universes G goes to infinity for the moment of the vanishing of the cyclically evolving scale factor a (the big crunch singularity), in the other universe G vanishes together with the scale factor a (equations (5.37) and (5.38)). In both cases the geometric evolution of the universes is the same while their physical evolution (the variability of the dynamical fundamental constants) differs.

4.1.4 Quantum entanglement in cyclic multiverse

For the concept of a multiverse to be considered as a physical theory it has to satisfy the condition of being falsifiable. This motivates us to introduce a mechanism of interaction of the universes that make up the multiverse. Such a mechanism can be introduced in cosmological models based on canonical quantum gravity. According to the postulates of quantum mechanics the space of states of a system composed of two quantum mechanical systems is the tensor product of Hilbert spaces associated with each of these systems. The elementary properties of the tensor product imply that most states which describe a composed quantum mechanical system are non-separable states that represent the so-called entangled states (separable states make a subset of the zero measure of the space resulting from tensor product). This leads to the conclusion that quantum entanglement is a generic phenomenon and it should take place in the multiverse (or at least such phenomena can not be excluded). In paper [$SA7$] we consider a multiverse consisting of two universes, each of them being treated as a quantum mechanical system described by the Wheeler-DeWitt equation (equation (20) in [$SA7$]):

$$\ddot{\phi} + \omega^2 \phi = 0, \tag{53}$$

where $\phi \equiv \phi(a)$ is the wave function of a single universe and the dot denotes the differentiation with respect to time. The time parameter for the single universe on the level of Wheeler-DeWitt equation is the scale factor a while the classical evolution of both universes is cyclic: sinusoidal (equation (4) in [$SA7$])

$$a(t) = a_0 \left| \sin\left(\pi \frac{t}{t_c}\right) \right|, \tag{54}$$

or tangential (equation (11) in [$SA7$])

$$a(t) = a_0 \left| \tan\left(\pi \frac{t}{t_s}\right) \right|, \tag{55}$$

depending on the assumed scenario. The function $\omega^2(a)$ plays the role of the Wheeler-DeWitt potential which in the sinusoidal scenario is given by (equation (18) in [$SA7$]):

$$\omega^2_{\sin}(a) \equiv a^2 - \Lambda a^4. \tag{56}$$

while in the tangential scenario reads as (equation (19) in [$SA7$]):

$$\omega^2_{\tan}(a) \equiv \Lambda^2 a^6 + 2\Lambda a^4 + a^2. \tag{57}$$

The space of states of such a multiverse is therefore represented by the tensor product of the two spaces of states associated with both universes. We then postulate that the quantum state of the multiverse is represented by the entangled state. The Hilbert spaces associated with individual universes were obtained by using the so-called third quantization procedure which is based on the formal similarity between the Wheeler-DeWitt and the Klein-Gordon equations [49, 50]. The role of the Klein-Gordon field is played by the wave function in the Wheeler-DeWitt equation, which as a result of the third quantization procedure becomes an operator acting on the Hilbert space. The third quantization itself is completely analogous to the quantization of the Klein-Gordon field. The result of the third quantization is the Hilbert space spanned by an orthonormal basis whose elements represent the occupation with universes which properties are determined by appropriate quantum numbers (these usually are the momentums in the minisuperspace). The third quantization procedure (as well as the quantization procedure of Klein-Gordon's field in time-dependent spacetimes) is an ambiguous one since it does not determine the vacuum state in an unique way. From the physical point of view, however, we are interested in only two representations out of infinitely many. One of these is an invariant representation that defines the vacuum state of the multiverse as independent of the classical evolution. In the other representation, the creation and annihilation operators associated with the orthonormal basis which spans the Hilbert space create or annihilate universes that evolve sinusoidally or tangentially (such universes have well-defined values of the momentum on the minisuperspace). Both representations are related with each other by means of the Bogoliubov's transformation (formulas (44) and (45) in [$SA7$]):

$$\hat{c}_- = \alpha \hat{b}_- - \beta \hat{b}^\dagger_+, \tag{58}$$

$$\hat{c}^\dagger_- = \alpha^* \hat{b}^\dagger_- - \beta^* \hat{b}_+, \tag{59}$$

where α and β are the Bogoliubov coefficients which satisfy the normalisation condition $|\alpha|^2 - |\beta|^2 = 1$, $\hat{c}^\dagger$ and $\hat{c}$ are the creation and annihilation operators of the invariant representation while $\hat{b}^\dagger$ and $\hat{b}$ are the operators that creates or annihilates universes evolving sinusoidally or tangentially (depending on the assumed scenario).

We consider a scenario [$SA7$] in which the state of the multiverse is represented by the vacuum state of the invariant representation. From the point of view of the other (abovementioned) representation such a state describes, however, the superposition of states with each state representing an entangled pair of universes with well-defined opposite values of the momentum on the minisuperspace, evolving sinusoidally or tangentially - depending on the assumed cyclic scenario (formula (43) in [$SA7$]):

$$|0_+ 0_-\rangle_c = \frac{1}{|\alpha|} \sum_{n=0}^{\infty} \left(\frac{|\beta|}{|\alpha|} \right)^n |n_-, n_+\rangle_b, \tag{60}$$

here the states $|n_-, n_+\rangle_b$ represent the entangled pairs of the universes with well-defined momentum, evolving accordingly to the assumed cyclic scenario. The coefficients which appear in the superposition (60) depend on the Bogoliubov coefficients that connect both representations. In the considered scenario the multiverse is all the time described by a pure state while the state of the single universe is given by the mixed state represented by the trace of the density matrix for the multiverse, calculated in the basis of the partner universe (formula (46) in [$SA7$]):

$$\rho_- = \mathrm{Tr}_+ \rho \equiv \sum_{n=0}^{\infty} {}_b\langle n_+ | \rho | n_+ \rangle_b, \tag{61}$$

where

$$\rho = |0_+ 0_-\rangle_c \langle 0_+ 0_-| = \frac{1}{|\alpha|^2} \sum_{n,m} \left(\frac{|\beta|}{|\alpha|} \right)^{n+m} |n_-, n_+\rangle_b \langle m_-, m_+|.$$

The tracing procedure gives the reduced density matrix which represents the Boltzmann equilibrium state (formula (48) in $[SA7]$):

$$\begin{aligned} \rho_- &= \frac{1}{|\alpha|^2} \sum_{n,m,l} \left(\frac{|\beta|}{|\alpha|} \right)^{n+m} \langle l_+|m_+\rangle |n_-\rangle_b \langle n_-| \langle m_+|l_+\rangle \\ &= \frac{1}{|\alpha|^2} \sum_{n} \left(\frac{|\beta|}{|\alpha|} \right)^{2n} |n_-\rangle_b \langle n_-| \\ &= \frac{1}{|\alpha||\beta|} \sum_{n} \left(\frac{|\beta|}{|\alpha|} \right)^{2n+1} |n_-\rangle_b \langle n_-| \\ &= \frac{1}{Z} \sum_{n} e^{-\frac{\omega}{T}(n+\frac{1}{2})} |n_-\rangle_b \langle n_-|, \end{aligned} \tag{62}$$

where $Z^{-1} = 2 \sinh \frac{\omega}{2T}$. The temperature associated with the state ρ_- is:

$$T \equiv T(a) = \frac{\omega(a)}{2 \ln \coth r}, \tag{63}$$

where

$$\tanh r \equiv \frac{|\beta|}{|\alpha|}. \tag{64}$$

The parameter r in the formula above characterises the entanglement of the pair of the universes. We also calculate the entropy of the entanglement resulting from the fact that individual universes are described by mixed states (equations (52) and (53) in $[SA7]$).

$$S(\rho) = -\mathrm{Tr}\,(\rho_- \ln \rho_-) = \cosh^2 \mathrm{r}\, \ln \cosh^2 \mathrm{r} - \sinh^2 \mathrm{r}\, \ln \sinh^2 \mathrm{r}. \tag{65}$$

In the case of the sinusoidally evolving model the entropy reaches the maximum value for both the minimum and the maximum value of the scale factor (entanglement entropy can be used as a measure of the entanglement strength) (Figure 7). In the case of the tangentially evolving model the entanglement entropy becomes infinite at the minimum value of the scale factor (at the moment of big bang singularity) and then monotonically decreases and reaches zero at the moment of the big rip singularity (Figure 8). Above results suggest an existence of a duality associated with the evolution of the entropy of entanglement. While in the sinusoidal model the entropy of entanglement is maximal (infinite) for the moment of maximal expansion in the tangential model the entropy of entanglement vanishes at the moment of the infinite expansion. On the other hand, the quantum effects in individual universes play an important role both at the moment of minimum and maximum (or infinite) expansion. This is due to the fact that both moments of the evolution (which corresponds to the minimum and maximum (or infinite) expansion) occur inside classically inaccessible regions.

We have seen that the Wheeler-DeWitt equation which describes individual universes is formally the same as the equation the describes the classical harmonic oscillator with time-dependent frequency $\omega(a)$, eq. (53). The energy of entanglement (determined by the value of r that describes the strength of entanglement) (formula (76) in $[SA7]$) given by

$$E_- = \frac{\omega_{\rm eff}}{2} = \omega \left(\sinh^2 r + \frac{1}{2} \right) . \tag{66}$$

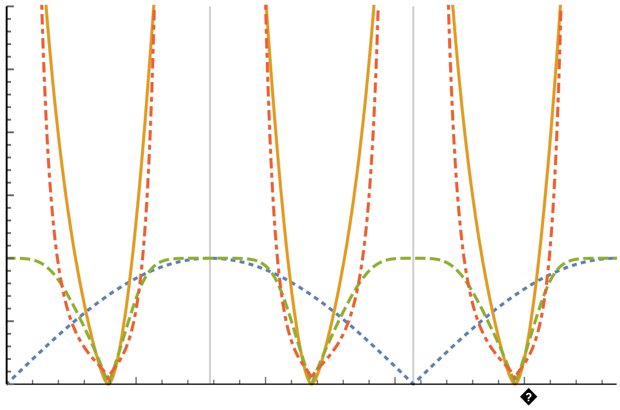

Figure 7: Scale factor (blue dashed), parameter $q \equiv \tanh r$ (green dashed), entropy of entanglement (yellow) and temperature of entanglement (red dashed) in the sinusoidal model. During the evolution the entanglement decreases, then it reaches the minimum value, and then it grows to attain its maximal value at the moment of maximal expansion.

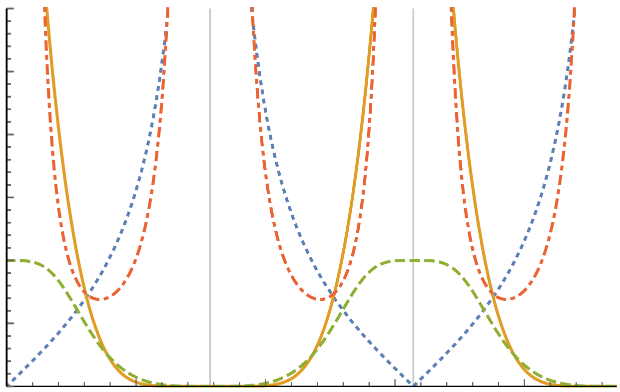

Figure 8: The scale factor (blue dashed), parametr $q \equiv \tanh r$ (green dashed), entropy of entanglement (yellow) and temperature of entanglement (red dashed) in the tangentially evolving model.

is, for vanishing entanglement (which corresponds to $r = 0$), identical with the ground state energy of the quantized oscillator associated with the frequency $\omega(a)$. The correction to the energy of entanglement associated with the non-vanishing entanglement ($r \neq 0$) can in principle be interpreted as the correction to the frequency of the classical oscillator. Adopting such an interpretation would require introduction of appropriate corrections in the Friedmann equation which describes the evolution of individual universes (formula (77) in [$SA7$]):

$$\frac{da}{dt} = \frac{\omega_{\text{eff}}}{a} = \frac{\omega}{a}\left(1 + 2\sinh^2 r\right) \tag{67}$$

Consequently, we obtain a model in which the classical evolution of individual universes making up the multiverse is corrected and the magnitude of this correction is related with the strength of the entanglement.

4.1.5 Conclusions

Despite the lack of experimental and observational evidence for the variability of the fundamental constants [51, 52, 53, 54, 55, 56, 57, 58] studying consequences of such variability seems to be meaningful. An attempt to construct models allowing variability of fundamental constants demands reconsidering the meaning and the role of the assumptions underlying the fundamental theories of physics. The formulation of models with varying speed of light is associated with many conceptual difficulties since it violates the foundations most of the fundamental physical models rest on.

The presented aspects of cosmological models which assume the variability of the speed of light indicate that such assumption may influence the interpretation of the supernovae type Ia and the redshift drift observational data. I also show that quantum models with varying speed of light and varying gravitational constant may lead to interesting scenarios of quantum cosmogenesis. I have demonstrated that within such models the cyclic evolution may occur with the regularised strong singularities. In addition, I found a cyclic model of the doubleverse (a multiverse containing two universes) where the loss of the entropy in one of the universes is compensated by the increase of the entropy in the other universe - a scenario consistent with second law of thermodynamics. I also introduced the mechanism of the interaction of the universes that make up the multiverse based on the phenomenon of quantum entanglement. I also investigated the consequences of assuming spatial dependence of the speed of light in the framework of the inhomogeneous cosmological Stephani models and examined if such dependence could in principle be detectable through future cosmological observations.

It should be stressed that the presented investigations focus only on a few selected phenomena and omit many cosmological aspects of very fundamental meaning (such as the evolution of perturbations or the growth of structures in the Universe). However, understanding of all the consequences of the assumption that speed of light may vary goes far beyond the scope of a single research project. An exhaustive description of the physical reality within any model based on the assumption of varying fundamental constants can only be developed by synthesising the results obtained by many researchers working on the problem of variability of fundamental constants.

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5 Scientific achievement not directly included in the habilitation procedure

5.1 Scientific publications in journals listed in Journal Citation Reports (JRC) database published after Ph.D. completion

SB1. A. Balcerzak, M.P. Dąbrowski, *"Brane $f(R)$ gravity cosmologies"*, Physical Review D81, 123527 (2010).

In this paper we investigate the cosmological evolution of the universe modeled in the framework of higher-order brane gravity theory with the action depending on some curvature invariants. We used the results published in papers [$SC2$] and [$SC3$] (the papers being a basis of the doctoral dissertation). My contribution consisted in: performing all the calculations, discussing the obtained results and giving active contribution to writing the article.

My percentage contribution is estimated at about 70%.
Impact Factor: 4.964 (2010 JCR).

SB2. A. Balcerzak, M.P. Dąbrowski, *"Randall-Sundrum limit of $f(R)$ brane-world models"*, Physical Review D85 (2011).

The paper is a continuation of the problems discussed in paper SB[1]. We investigate the relation of the higher-order gravity brane cosmological model with the standard Randall-Sundrum brane model. We show that the higher-order gravity brane model transforms in the limit into the standard Randall-Sundrum brane model. My contribution consisted in: performing most of the calculations, discussing the obtained results and giving active contribution to writing the article.

My percentage contribution is estimated at about 60%.
Impact Factor: 4.558 (2011 JCR).

SB3. A. Balcerzak, T. Denkiewicz, *"Density perturbations in a finite scale factor singularity universe"*, Physical Review D 86, 023522 (2012).

We study the growth of cosmological perturbations (the growth of the matter density contrast) in a cosmological scenario containing the sudden future singularity. We investigate the possibility of explaining of the large-scale structure data within the cosmological models with a sudden future singularity in which the dark matter perturbations evolve independently from the perturbations in the dark energy. My contribution consisted in: performing part of the numerical calculations, discussing the methods and the results obtained, and giving active contribution to writing the article.

My percentage contribution is estimated at about 50%.
Impact Factor: 4.691 (2012 JCR).

SB4. A. Balcerzak, M. P. Dąbrowski, *"Redshift drift in a pressure-gradient cosmology"*, Physical Review D87, 063506 (2013).

In this paper we derive the formula for the time evolution of the redshift (the redshift drift) of cosmological sources for the spherically symmetric Stephani model. We show that the redshift drift curve for the Stephani model differs significantly from the redshift drift curve for the LTB and the ΛCDM models. We also argue that the due to this difference the future observations of the redshift dirft may be used to differentiate between the mentioned models. My contribution consisted in: performing all the calculations, discussing the obtained results and giving active contribution to writing the article.

My percentage contribution is estimated at about 70%.
Impact Factor: 4.864 (2013 JCR).

SB5. A. Balcerzak, M.P. Dąbrowski, T. Denkiewicz, *"Off-center observers versus supernovae in inhomogeneous pressure universes"*, Astrophysical Journal 792, 92-99 (2014).

In this paper we put constraints on the the position of the observer in the universe described by the spherically symmetric Stephani metric by using the observed positions and the luminosity distances of supernovae type Ia from Union2 557 set. My contribution consisted in: performing part of the numerical calculations, discussing the methods and the results obtained, and giving active contribution to writing the article.

My percentage contribution is estimated at about 50%.
Impact Factor: 5.593 (2014 JCR).

SB6. A. Balcerzak, M.P. Dąbrowski, T. Denkiewicz, D. Polarski, D. Puy, *"A critical assessment of some inhomogeneous pressure Stephani models"*, Physical Review D91, 083506 (2015).

In this paper we fit the spherically symmetric Stephani model with an observer placed at the center of symmetry to the supernovae type Ia observational data, the shift parameter of the cosmic microwave background and the barionic acoustic oscillations. We also show that the model that fits to the abovementioned observational data reproduces the temporal evolution of the redshift (the redshift drift) of the ΛCDM model, which indicates that there exists a degeneracy with respect to the mentioned observational tests. My contribution consisted in: performing part of the calculations, discussing the methods and the results obtained, and giving active contribution to writing the article.

My percentage contribution is estimated at about 50%.
Impact Factor: 4.506 (2015 JCR).

SB7. K. Marosek, A. Balcerzak, *"Strength of singularities in varying constants theories"*, European Physical Journal C 79, 287 (2019).

In this paper we consider a specific type of the bimetric theory of gravitation with the two different metrics introduced in the cosmological frame. Both metrics respect all the symmetries of the standard FLRW solution and contain conformally related spatial parts. One of the metric is assumed to describe the causal structure for the matter. Another metric defines the causal structure for the gravitational interactions. A crucial point is that the spatial part of the metric describing gravity is given by the spatial part of the matter metric conformally rescaled by a time-dependent factor α which, as it turns out, can be linked to the effective gravitational constant and the effective speed of light. In the context of such a bimetric framework we examine the strength of some singular cosmological scenarios in the sense of the criteria introduced by Tipler and Królak. In particular, we show that for the nonsingular scale factor associated with the matter metric, both the vanishing or blowing up of the factor α for some particular moment of the cosmic expansion may lead to a strong singularity with infinite value of the energy density and infinite value of the pressure. My contribution consisted in: performing part of the calculations, discussing the methods and the results obtained, and giving active contribution to writing the article.

My percentage contribution is estimated at about 50%.
Impact Factor: 5.172 (2017 JCR).

5.2 Scientific publications in journals listed in Journal Citation Reports (JRC) database published before Ph.D. completion

SC1. A. Balcerzak, M.P. Dąbrowski, *"Strings at Future Singularities"*, Physical Review D73, 101301 (2006) (R).

We investigate the evolution of the cosmic string in the Friedmann spacetime with the Sudden Future Singularity and the Big Rip singularity. In particular we examine the passing of the cosmic string throughout both singularities. We also classify these singularities by using the criterion of Tipler and Królak. My contribution consisted in: performing half of the calculations, discussing the results obtained, and giving active contribution to writing the article.

My percentage contribution is estimated at about 50%.
Impact Factor: 4.896 (2006 JCR).

SC2. A. Balcerzak, M.P. Dąbrowski, *"Generalized Israel Junction Conditions for a Fourth-Order Brane World"*, Physical Review D77, 023524 (2008).

This work explores the possibility of formulating the brane gravity models within the framework of the higher-order gravity with action depending on some curvature invariants. We extend the Israel formalism of the Einstein's general relativity theory since it produces improperly defined terms of the distributional type while inserting the brane into the spacetime with geometry governed by the higher-order gravity theory. We formulate the conditions for the existence of an infinitely thin layer of matter (the brane) in spacetime of higher-order gravity theory in terms of the generalized Israel junction conditions. My contribution consisted in: performing most of the calculations, discussing the methods and the results obtained, and giving active contribution to writing the article.

My percentage contribution is estimated at about 70%.
Impact Factor: 5.050 (2008 JCR).

SC3. A. Balcerzak, M.P. Dąbrowski, *"Gibbons-Hawking Boundary Terms and Junction Conditions for Higher-Order Brane Gravity Models"*, Journal of Cosmology and Astroparticle Physics (JCAP) 01, 018 (2009).

In this paper we formulate the general conditions for the existence of the brane (the generalized Israel junction conditions) by using the variational formalism with Gibbons-Hawking boundary terms in the higher-order gravity theory with action depending on some curvature invariants. We derive the explicit form of the boundary terms that lead to the generalized Israel junction conditions. My contribution consisted in: performing most of the calculations, discussing the methods and the results obtained, and giving active contribution to writing the article.

My percentage contribution is estimated at about 70%.
Impact Factor: 6.502 (2009 JCR).

5.3 Conference Proceedings

SD1. M. P. Dąbrowski, A. Balcerzak, *"Big-Rip, Sudden Future, and Other Exotic Singularities in the Universe"*, H. Kleinert, R.T. Jantzen and R. Ruffini (eds.) (World Scientific, Singapore, 2008), 2051-2053. Proceedings "Proceedings of the Eleventh Marcel Grossmann Meeting on General Relativity", Berlin, Germany, July 23 - 29, 2006.

SD2. M.P. Dąbrowski, A. Balcerzak, *"Higher-order brane gravity models"*, AIP Conference Proceedings Volume 1241 (Melville, New York, 2010), 468-476. Proceedings "Invisible universe", Paris, France, 29.06-3.07.2009.

SD3. A. Balcerzak, *"Fourth-order braneworld gravity"*, Annalen der Physik (Berlin) 19 (2010), 271-275. Proceedings "Grassmannian Conference in Fundamental Cosmology", Szczecin, Poland, September 14-19, 2009.

SD4. A. Balcerzak, *"Redshift drift and inhomogeneities"*, AIP Conference Proceedings Volume 1514 (Melville, New York, 2013), 35-38. Proceedings "Multiverse and Fundamental Cosmology", Szczecin, Poland, September 10-14, 2012.

SD5. M.P. Dąbrowski, K. Marosek, A, Balcerzak, *"Standard and exotic singularities regularized by varying constants"*, Memoria della Societa Astronomica Italiana 85, 44-49 (2014). Proceedings "Varying fundamental constants and dynamical dark energy", Sesto, Italy, July 8-12, 2013.

SD6. M.P. Dąbrowski, V. Salzano, A. Balcerzak, R. Lazkoz, *"New tests of variability of the speed of light"*, EPJ Web Conf. 126 (2016) 04012. Proceedings "4^{th} International Conference on New Frontiers in Physics (ICNFP 2015)", Crete, Greece, August 23-30, 2015.

5.4 Bibliometric Summary

5.4.1 Impact Factor

Impact Factor for the year of publication (for 2017-papers we have used 2016 numbers) and score from MNiSW according to list A from 2016.

	Impact Factor JCR	Score MNiSW
Papers entering the habilitation	34.8	260
All publications	85.596	625

5.4.2 Citations

Number of citations at 20 March 2019.

	Web of Science	NASA ADS	inSpire	Google Scholar
Total Citations	181	221	225	277
Total Citations excluding self-citations	167	200	167	–
H-index	8	9	9	10

5.5 Participation in research projects

1. Contractor in the grant founded by National Science Centre: "Cosmologies with weak singularities in fundamental theories and the Multiverse" Nr N N202 3269 40 (2011-2013).
2. Contractor in the grant founded by National Science Centre: "New consequences of the variability of fundamental constants in physics and cosmology" Maestro-3 (DEC-2012/06/A/ST2/00395) (2013-2018).

5.6 International and National Prizes for Scientific Activity

06.2016 West Pomeranian Nobel 2015 in Fundamental Science, a prestigious award conferred by the West Pomeranian Leader of Science (Zachodniopomorski Klub Liderów Nauki - ZKLN) for my research devoted to cosmological singularities.

5.7 Seminars held in national and international scientific institutions

12.2008 Talk on the "Secondo TRR33 Winter School", Passo Tonale, Italy, "Junction conditions for Higher-Order Brane Gravity Theories";

01.2009 Talk for the Seminar of Exact Results of Quantum Theory and Gravitation, Institute of Theoretical Physics, University of Warsaw, "Israel junction conditions in fourth-order gravity theory for brane universes";

05.2009 Talk for the Seminar of the Institute of Theoretical Physics, University of Lodz, "Higher-order brane gravity theories";

09.2009 Talk on the "Grassmannian Conference in Fundamental Cosmology" conference, Szczecin, Poland, "Fourth-order braneworld gravity";

12.2010 Talk on the "Montpellier Cosmology Workshop 2010", Montpellier, France, "Higher-order brane universes";

09.2011 Talk on XLI Meeting of Polish Physicists, Lublin, Poland, "Fourth-order braneworld gravity";

10.2011 Talk for the Seminar "Cosmology and Particles", Institute of Theoretical Physics, University of Warsaw, "Higher-order brane gravity";

09.2012 Talk on the “Multiverse and Fundamental Cosmology (Multicosmofun’12)” conference, Szczecin, Poland, “Redshift drift and inhomogeneities”;

11.2014 Talk on the “Dark Side of the Universe 2014” conference, Cape Town, Republic of South Africa, “A critical assessment of some inhomogeneous pressure Stephani models”;

01.2015 Talk for the Seminar of the Institute of Physics, University od Szczecin, “Inhomogeneous pressure Stephani models”;

09.2016 Talk on the “Varying Constants and Fundamental Cosmology VARCOSMOFUN’16” conference, Szczecin, Poland, “Non-minimally coupled varying constants quantum cosmologies”;

12.2018 Talk for the Seminar of the Szczecin Cosmology Group, Institute of Physics, University od Szczecin, “Nature of quantum entanglement”;

6 Teaching and popularization of science

6.1 Teaching

6.1.1 University Lectures

2015-2018 Semester classes for students of physics at the Faculty of Mathematics and Physics of the University of Szczecin (bachelor and master studies) conducted as part of the pensum. Subjects: *General physics, Quantum mechanics, Quantum mechanics II, Numerical methods, Nuclear and particle physics, Thermodynamics and statistical physics, Statistical physics, Theoretical mechanics, Fluid mechanic, Introduction to mathematics, Physics Laboratory I, Biophysics, Introduction to condensed matter physics*

6.1.2 Supervision of Ph.D. Students

2012-2017 Co-supervisor of the Ph.D. student Konrad Marosek (Supervisor - Prof. Mariusz P. Dąbrowski), Institute of Physics, University of Szczecin. Title of the thesis: “Regularising cosmological singularities and cyclic multiverses in theory with dynamical fundamental constants” (defense on 22 November 2017).

6.2 Popularization of science

2009-2012 Conducting classes popularising physics for junior high school students as part of the project “Z fizyką, matematyką i przedsiębiorczością zdobywamy świat” organized by the University of Szczecin and COMBIDATA Poland (The project was co-financed by European Union).

6.3 Organisation of scientific conferences

09.2019	Chair of the Local Organizing Committee of the 6th Conference of the Polish Society on Relativity POTOR-6 which will be held in Szczecin on $23-26$ September.
09.2016	Member of the Local Organizing Committee of the "Varying Constants and Fundamental Cosmology - VARCOSMOFUN'16" conference held is Szczecin on $12-17$ September.
09.2012	Member of the Local Organizing Committee of the "Multiverse and Fundamental Cosmology - Multicosmofun'12" conference held is Szczecin on $10-14$ September.
09.2009	Member of the Local Organizing Committee of the "Grassmannian Conference in Fundamental Cosmology - Grasscosmofun'09" conference held is Szczecin on $14-19$ September .

6.4 Reviewer work

- Reviewer of the articles in the Physical Review D journal

Adam Balcerzak