Beyond Quantum Mechanics

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Quantum Mechanics

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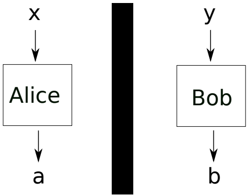
- Because 1. and 2. rather than 'improve' quantum mechanics, try to understand how 3. is possible

Intrinsic randomness

- How do we prove that quantum mechanics is intrinsically random? (e.g. that we can generate a ‘truly’ random sequence)



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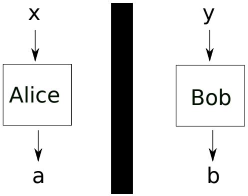

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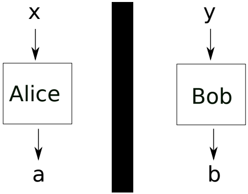

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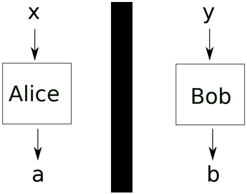

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- Local hidden-variable model

$$p(a, b|x, y, \lambda) = p(\lambda)p(a|x, \lambda)p(b|y, \lambda).$$

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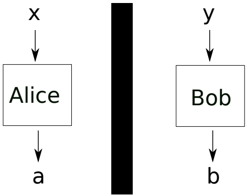

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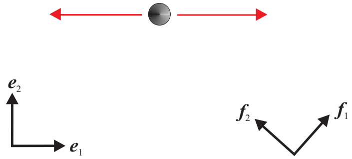
- Bell inequalities, fulfilled by all deterministic (=local hidden variables) theories.

$$\sum_{a,b,x,y} \alpha_{ab}^{xy}\, p(a, b|x, y) \leq \mathcal{S}_L,$$

EPR scheme

- Spin component (e_i, f_j) measurements $(1, -1)$ of two products of decayed spin 0 particle



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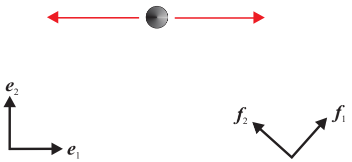

- Correlations:

$$\langle e_i f_j \rangle = \sum_{a,b=\pm 1} a \cdot b \cdot p(a, b | e_i, f_j)$$

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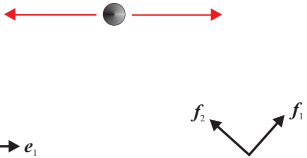

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- Classically: $S \leq 2$

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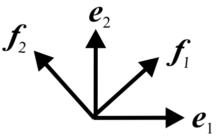

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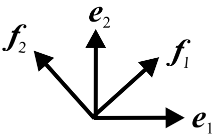

- $S = 2\sqrt{2}$

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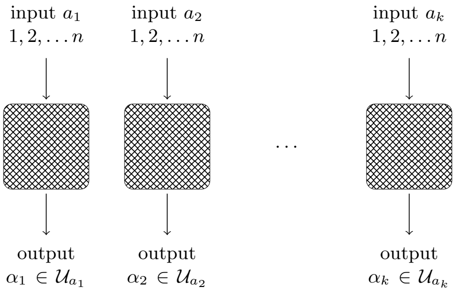
- Rather than try to close the loop, try to understand why the intrinsic randomness is possible

No-signaling boxes



- $P(\alpha_1\alpha_2 \ldots \alpha_k | a_1 a_2 \ldots a_k)$ probability of an outcome $(\alpha_1, \alpha_2, \ldots, \alpha_k)$ given an input $(a_1, a_2, \ldots, a_k)$

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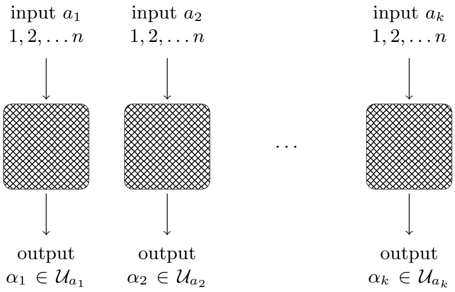

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- positive, normalized, and *no-signaling*

$$\sum_{\alpha_i} P(\alpha_1\ldots\alpha_i\ldots\alpha_k|a_1\ldots a_i\ldots a_k) = \sum_{\beta_i} P(\alpha_1\ldots\beta_i\ldots\alpha_k|a_1\ldots b_i\ldots a_k),$$

i.e. changing the input in one box does not influence the outcomes of other ones

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- Classical and quantum physics restrict S further

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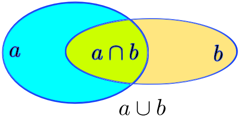

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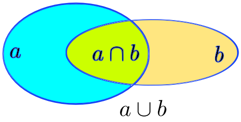

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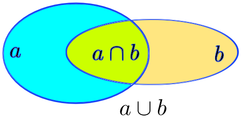

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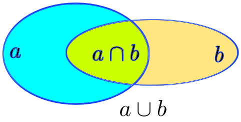

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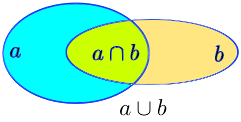

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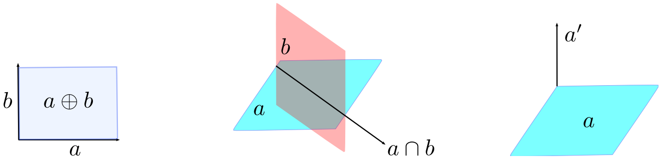

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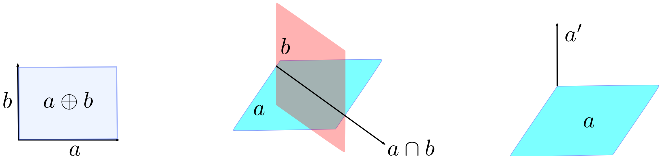

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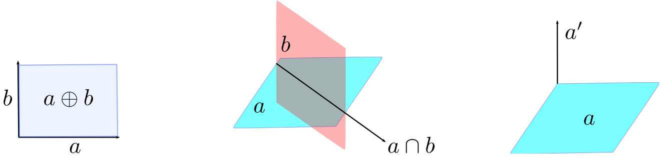

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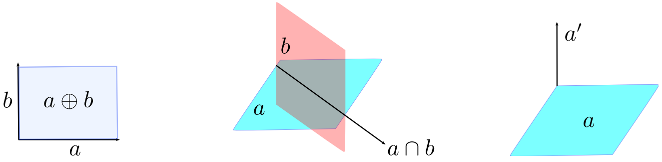

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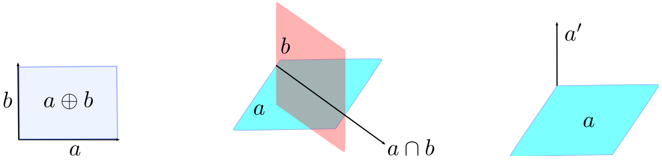

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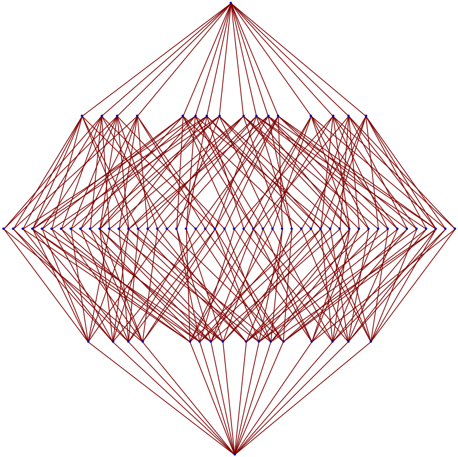
Hasse diagram



$a \leq b$ iff $a = a \wedge b$

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  Conclusion: **no-signaling boxes are no competitor to quantum mechanics when it comes to possible ‘intrinsic’ randomness.**

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- Causal structure in QM and other theories (in QM one can condition the causal order by the theory itself)