What ξ? Cosmological constraints on the non-minimal coupling constant

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ABSTRACT

In dynamical system describing evolution of universe with the flat Friedmann–Robertson–Walker symmetry filled with barotropic dust matter and non-minimally coupled scalar field with a constant potential function an invariant manifold of the de Sitter state is used to obtain exact solutions of the reduced dynamics. Using observational data coming from distant supernovae type Ia, the Hubble function $H(z)$ measurements and information coming from the Alcock–Paczyński test we find cosmological constraints on the non-minimal coupling constant ξ between the scalar curvature and the scalar field. For all investigated models we can exclude negative values of this parameter at the 68% confidence level. We obtain constraints on the non-minimal coupling constant consistent with condition for conformal coupling of the scalar field in higher dimensional theories of gravity.

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Cosmology

General Theory of Relativity

$$S = \frac{1}{2\kappa^2}\int \mathrm{d}^4x\sqrt{-g}\left(R - 2\Lambda\right) + S_m$$

Friedmann-Robertson-Walker symmetry

$$\left(\frac{H}{H_0}\right)^2 = \Omega_{\Lambda,0} + \Omega_{m,0}\left(\frac{a}{a_0}\right)^{-3} + \Omega_{r,0}\left(\frac{a}{a_0}\right)^{-4} + \dots$$

$$S_T = S - \frac{1}{2}\int \mathrm{d}^4x\sqrt{-g}\left(\nabla^\alpha\phi\,\nabla_\alpha\phi + 2U(\phi)\right)$$

- inflationary epoch – inflaton
- current accelerated expansion – quintessence

Working cosmological model:

"BB" $\rightarrow$ Inflation $\rightarrow$ RDE $\rightarrow$ MDE $\rightarrow$ de Sitter

Dynamical system theory:

an unstable node $\rightarrow$ a saddle $\rightarrow$ a saddle $\rightarrow$ a saddle $\rightarrow$ a stable node/attractor

or without BB singularity

an unstable node $\rightarrow$ a saddle $\rightarrow$ a saddle $\rightarrow$ a stable node/attractor

The theory

We start from the total action of the theory

$$S = S_g + S_\phi + S_m \, ,$$

consisting of the gravitational part described by the standard Einstein-Hilbert action integral

$$S_g = \frac{1}{2\kappa^2} \int \mathrm{d}^4x \sqrt{-g}\, R \, ,$$

where $\kappa^2 = 8\pi G$, and the matter part of the theory is composed of two substances. One is in the form of non-minimally coupled scalar field

$$S_\phi = -\frac{1}{2} \int \mathrm{d}^4x \sqrt{-g} \Big(\varepsilon \nabla^\alpha \phi \, \nabla_\alpha \phi + \varepsilon \xi R \phi^2 + 2U(\phi) \Big) \, ,$$

where $\varepsilon = +1, -1$ corresponds to the canonical and the phantom scalar field, respectively, and the second one in the form of barotropic matter

$$S_m = \int \mathrm{d}^4x \sqrt{-g} \mathcal{L}_m \, .$$

Asymptotically monomial potential functions

Inflationary paradigm:

"plateau-like" potential functions $\frac{U'(\phi)}{U(\phi)} \to 0$ as $\phi \to \infty$

Our assumptions:

$\lambda = -\phi\frac{U'(\phi)}{U(\phi)} \to$ const., not only potential function with $U(\phi) \to$ const. as $\phi \to \infty$ but all possible potential functions with an asymptotic behaviour $U(\phi) \propto \phi^{\alpha}$

Projective coordinates and analysis at infinity

projective coordinates

$$u \equiv \frac{x}{z} = \frac{\dot{\phi}}{H\phi}, \quad \bar{v} \equiv \frac{y^2}{z^2} = \frac{1}{2}\frac{U(\phi)}{H^2\phi^2}, \quad \bar{w} \equiv \frac{1}{z^2} = \frac{6}{\kappa^2}\frac{H_0^2}{H^2\phi^2},$$

u^*	$\bar{v}^*$	$\frac{\dot{H}}{H^2}\Big\|_*$
$-6\xi \pm \sqrt{-6\xi(1-6\xi)}$	0	$-2 + \frac{(u^*)^2}{6\xi}$
$-\frac{(4+\lambda)\xi}{1-(2-\lambda)\xi}$	$-\varepsilon\frac{(1-6\xi)\xi(6-(2-\lambda)(10+\lambda)\xi)}{(1-(2-\lambda)\xi)^2}$	$\frac{1}{2}\frac{(2+\lambda)(4+\lambda)\xi}{1-(2-\lambda)\xi}$

Observational constraints

Working assumptions:
barotropic dust matter + $U(\phi) = U_0 = \text{const.}$

the energy conservation condition, Friedmann equation

$$\left(\frac{H(a)}{H(a_0)}\right)^2 = \Omega_{\Lambda,0} + \Omega_{m,0}\left(\frac{a}{a_0}\right)^{-3} + \varepsilon(1-6\xi)x^2 + \varepsilon 6\xi(x+z)^2 \,,$$

where

$$\Omega_{m,0} \equiv \frac{\kappa^2 \rho_{m,0}}{3H_0^2}\,, \quad \Omega_{\Lambda,0} \equiv \frac{\kappa^2 U_0}{3H_0^2}\,.$$

Observational data:
Union2.1+H(z)+Alcock-Paczyński test

The general case

The Hubble function can be expanded in to the Taylor series

$$\left(\frac{H(a)}{H(a_0)}\right)^2 = h^2(a, \Omega_{bm,0}, \varepsilon, \xi, x_0, z_0)$$
$$\approx \Omega_{\Lambda,0} + \Omega_1 \left(\frac{a}{a_0}\right)^{-1} + \Omega_2 \left(\frac{a}{a_0}\right)^{-2} + \Omega_3 \left(\frac{a}{a_0}\right)^{-3} + \dots$$

where the density parameters are $\Omega_i = \Omega_i(\Omega_{bm,0}, \varepsilon, \xi, x_0, z_0)$. From the energy conservation condition we have

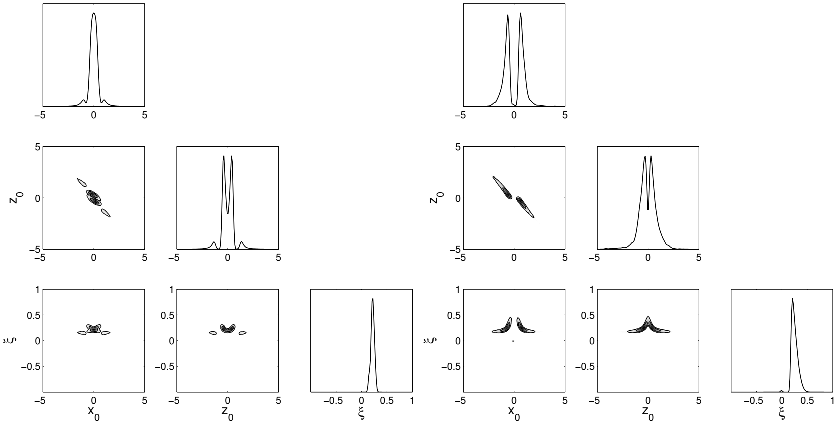
$$\Omega_1 + \Omega_2 + \Omega_3 + \dots = \Omega_{bm,0} + \varepsilon(1-6\xi)x_0^2 + \varepsilon 6\xi(x_0+z_0)^2 \, .$$

With the observational data used we can expect that $\Omega_i \approx 0$ for $i > 3$. Additionally, the ΛCDM model is favoured by the data and we can expect that $\Omega_1 \approx 0$ and $\Omega_2 \approx 0$. Thus we obtain that the leading term in the Taylor series above is the following

$$\Omega_3 \approx \Omega_{bm,0} + \varepsilon(1-6\xi)x_0^2 + \varepsilon 6\xi(x_0+z_0)^2 \, .$$

The last terms in this formula can be interpreted as an effective dark matter in the model resulting from the present evolution of the scalar field.

Observational constraints



Non-minimal coupling and a constant potential

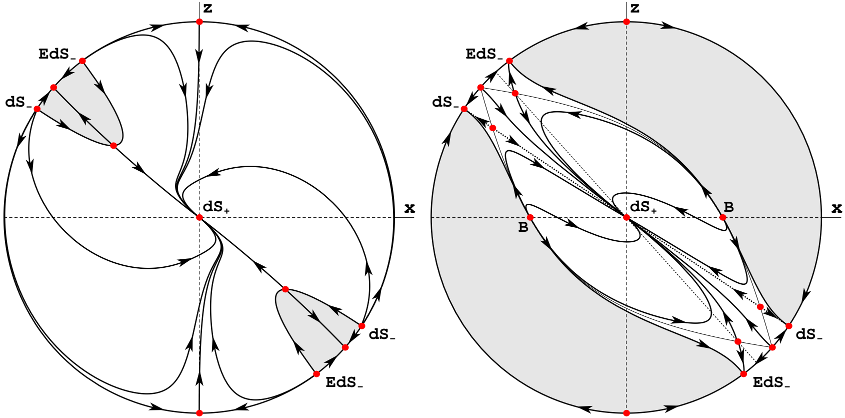


Figure: $\xi = \frac{3}{16}$: $\varepsilon = +1,\ U_0 > 0$ – left; $\varepsilon = -1\ U_0 > 0$ – right

An asymptotically quadratic potential function

Work in progress :
"A new generic cosmological scenario without singularity"

$$U(\phi) = \pm \frac{1}{2} m^2 \phi^2 \pm M^{4+n} \phi^{-n} \,,$$

where $n > -2$
an unstable de Sitter state

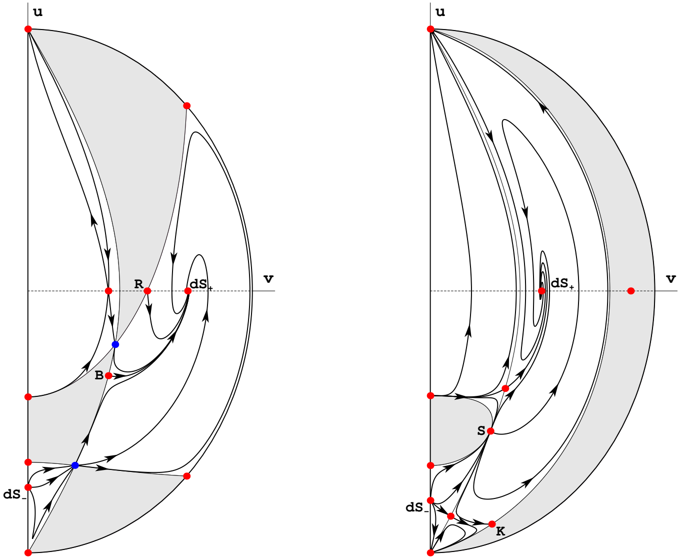
$$\frac{3}{16} < \xi < \frac{1}{4}$$

the energy conservation condition

$$\left(\frac{H(a)}{H(a_0)} \right)^2 \Big|_* = -\varepsilon \left(\pm \frac{m^2}{H_0^2} \right) \frac{(1-4\xi)^2}{2\xi(1-6\xi)(3-16\xi)} \,,$$

subject to condition $-\varepsilon \left(\pm \frac{m^2}{H_0^2} \right) > 0 \,.$

Non-minimally coupled scalar field



Conformal Invariance and $U(\phi) = U_0\phi^\alpha$

The Klein-Gordon equation and the conformal (Weyl) transformation

$$\tilde{\Box}\tilde{\phi} - \xi\tilde{R}\tilde{\phi} - \varepsilon\alpha U_0\tilde{\phi}^{\alpha-1} = \Omega^{-\frac{n+2}{2}}\left(\Box\phi - \xi R\phi - \varepsilon\alpha U_0\phi^{\alpha-1}\right) = 0\,.$$

$$\xi = \xi_{\rm conf} = \frac{1}{4}\frac{n-2}{n-1}\,, \qquad \alpha = \alpha_{\rm conf} = \frac{2n}{n-2}\,,$$

In this way we obtain a discrete set of theoretically allowed values of the non-minimal coupling constant suggested by the conformal invariance condition of the scalar field in $n \geq 2$ space-time dimensions:

$$\{(n,\xi,\alpha)\} = \Big\{(2,0,\infty)\,,\left(3,\frac{1}{8},6\right),\left(4,\frac{1}{6},4\right), \left(5,\frac{3}{16},\frac{10}{3}\right),\left(6,\frac{1}{5},3\right),\ldots,\left(\infty,\frac{1}{4},2\right)\Big\}\,.$$

Conclusions

Is conformal invariance the fundamental symmetry ?