High energy QCD and gluon saturation beyond leading power (of energy)

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High energy scattering in QCD

Quantum Chromodynamics (QCD)

The theory of strong interactions between **quarks** and **gluons** (fundamental particles that make up protons and neutrons in atomic nuclei).

Non-Abelian theory: gluons interact with each other.
Confinement: free quarks are never observed.
Asymptotic freedom: at high energies $\alpha_s \rightarrow 0$, enabling use of perturbation theory.

At high scattering energies ($\sqrt{s} \propto 1/x$) the wave function of the scattering objects are **dominated by the gluons.**

Experimental studies

HERA (1992-2007)/Relativistic Heavy Ion Collider (RHIC) (since 2000)/Large Hadron Collider (LHC) (since 2010).

The familiar "three quark" picture of the proton breaks down- the proton is a dense, dynamic sea of gluons.
source: www.bnl.gov

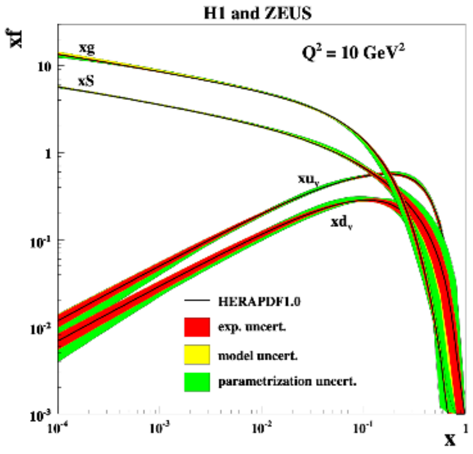
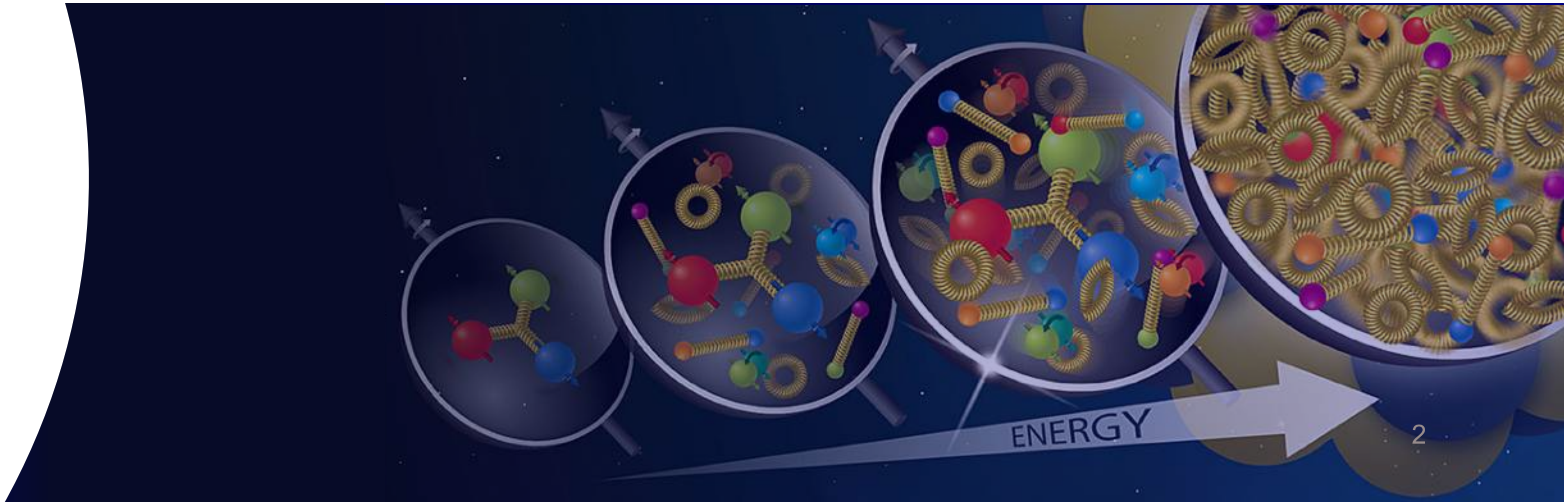
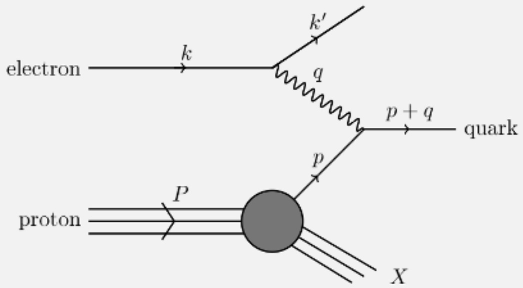


Figure taken from [arXiv:0911.0884 [hep-ex]]



High energy scattering in QCD (II)

DIS in QCD



Three Lorentz invariant quantities

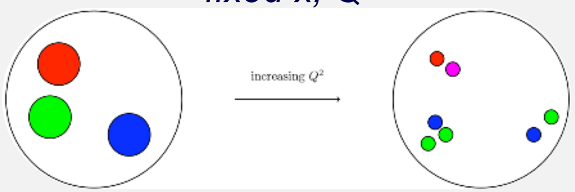
- $q^2 = -Q^2$ (virtuality of the incoming photon)
- $x = Q^2/(2P \cdot Q)$ (longitudinal momentum fraction carried by the parton)
- $s \simeq 2P \cdot Q$ (energy of the colliding $\gamma - p$ system)

Increasing the energy ($s = Q^2/x$) of the system

(Q is transverse resolution scale)

Bjorken Limit

fixed x, $Q^2 \to \infty$



Evolution wrt Q^2 — DGLAP equations

Regge-Gribov Limit

fixed Q^2, $x \to 0$

Evolution wrt $Y = \ln(1/x)$

linear BFKL   non-linear BK/JIMWLK

Gluon saturation and CGC

BFKL evolution: scattering probability grows without a bound and exceeds unity at $Y \simeq \frac{1}{\alpha_s}\ln(\frac{1}{\alpha_s})$ → violation of unitarity

AT HIGH DENSITY

linear BFKL → ignores gluon recombination | nonlinear BK/JIMWLK → includes gluon recombination (gluon density stops growing)

→ gluon saturation phenomenon

- saturation regime is characterized by Q_s
- measure of the strength of gluon interactions at high density
- $Q_s \gg \Lambda_{QCD} \Rightarrow$ weak coupling methods can still be applied

Effective theory to study gluon saturation - Color Glass Condensate (CGC)

It has been utilized to describe different aspects of RHIC and LHC data - quite successful!

Strong assumption: Eikonal approximation (keeping only leading power terms in energy)

→ good for asymptotically high energies!

Hints of gluon saturation regime seen at RHIC and LHC, but its first unambiguous evidence is expected to be seen at the Electron-Ion Collider (EIC)

A new era for QCD
the Electron-Ion Collider (EIC)

The new collider in USA, **$2 billion**, first run expected **early 2030s**

EIC User Group: consists of more than **1400 scientists** from over **290 institutions** from **38 countries** around the world

NCBJ QCD Theory group is active in EICUG community - members of the EIC Theory Alliance and organizer of earlier EICUG meeting (Warsaw June 2023)

www.bnl.gov/eic/science

Among main goals of EIC

Precision 3D imaging of protons and nuclei

The Electron-Ion Collider will take three-dimensional precision snapshots of the internal structure of protons and atomic nuclei. As they pierce through the larger particles, the high-energy electrons will interact with the internal microcosm to reveal unprecedented details—zooming in beyond the simplistic structure of three valence quarks bound by a mysterious force. (...)

Search for saturation

Capturing the dynamic action of gluons within protons and nuclei will give scientists a way to test their understanding of these particles' ephemeral properties. As gluons flit in and out of the vacuum, multiplying and recombining, scientists suspect they may reach a steady state of saturation called a "color glass condensate." (...)

Realization of the gluon saturation regime requires increasing the precision of the theory predictions!

EIC is planned to probe **lower collision energies (20-140 GeV)** compared to LHC (5-13 TeV).

To achieve precision for the theory predictions for the EIC physics power corrections in energy (subeikonal corrections) are of key importance.

CGC/Small-x group at NCBJ → 3 Permanents/2 Postdocs/2 PhD students → is amongst the strongest groups specialized on subeikonal studies in the world

Eikonal approximation within the CGC

Dense target is represented by strong semiclassical gluon field $\mathcal{A}^\mu_a(x) = O\left(1/g\right)$ at weak coupling g

High Energy limit can be achieved by boosting the target along x^-

$$\mathcal{A}^\mu_a(x) \mapsto \begin{cases} \gamma_t\, \mathcal{A}^-_a\left(\gamma_t\, x^+,\ \frac{x^-}{\gamma_t},\ \mathbf{x}\right) \\ \frac{1}{\gamma_t}\mathcal{A}^+_a\left(\gamma_t\, x^+,\ \frac{x^-}{\gamma_t},\ \mathbf{x}\right) \\ \mathcal{A}^i_a\left(\gamma_t\, x^+,\ \frac{x^-}{\gamma_t},\ \mathbf{x}\right) \end{cases}$$

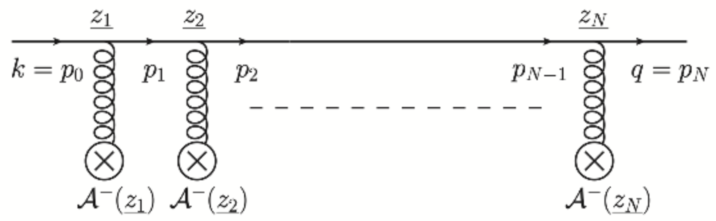
The eikonal approximation can be understood as the limit of infinite boost of $\mathcal{A}^\mu_a(x)$ →

- Background field is independent of x^- due to Lorentz time dilation **(static limit)**→ no p^+ transfer from the target
- Background field is Lorentz contracted **(shockwave limit)**→ no transverse motion within the target.
- Only the largest component of the background field is accounted for during the interaction.

In the Eikonal limit the background field →

$$\mathcal{A}^\mu(x^+, x^-, \mathbf{x}) \simeq \delta^{\mu -}\mathcal{A}^-_a(x^+\mathbf{x}) \propto \delta(x^+)$$

$\left(g\mathcal{A}^-(x^+, \mathbf{x})\right)^n$ are resumed to all orders → resummation leads to Wilson line encoding the multiple interactions



$$\mathcal{U}_F(x^+, y^+; \mathbf{z}) \equiv \mathcal{P}_+ \exp\left[-ig\int_{y^+}^{x^+} dz^+\, t\cdot\mathcal{A}^-(z^+, \mathbf{z})\right]$$

NEik corrections in the CGC
gluon background

Next-to_Eikonal (NEik) corrections are $\mathcal{O}(1/\gamma_t)$ at the level of the boosted background field

In pure gluon background, NEik corrections are from relaxing either of the three approximations

- Interactions with the suppressed components of background field (transverse component)
- Finite longitudinal width of the target — transverse motion of the parton in the medium
- x^- dependence of the background field beyond infinite Lorentz dilation

e.g., structures appearing in quark propagator from before to after the medium at NEik accuracy:

Generalized Eikonal Wilson line

$$\mathcal{U}_F(x^+,y^+;\mathbf{z},z^-) \equiv \mathbf{1} + \sum_{N=1}^{+\infty} \frac{1}{N!}\,\mathcal{P}_+\left[-ig\int_{y^+}^{x^+} dz^+\, t\cdot\mathcal{A}^-(z)\right]^N$$

NEik decorated Wilson lines

$$\mathcal{U}^{(1)}_{F;j}(\mathbf{z}) = -2\int dz^+\, z^+\, \mathcal{U}_F(+\infty,z^+;\mathbf{z})\,[-igt\cdot\mathcal{F}_j^{\ -}(z^+,\mathbf{z})]\,\mathcal{U}_F(z'^+,-\infty;\mathbf{z})$$

$$\mathcal{U}^{(2)}_{F}(\mathbf{z}) = \int dz^+\int dz'^+\,(z^+-z'^+)\,\theta(z^+-z'^+)\,\mathcal{U}_F(+\infty,z^+,\mathbf{z})\,[-igt\cdot\mathcal{F}_j^{\ -}(z^+,\mathbf{z})]\,\mathcal{U}_F(z^+,z'^+;\mathbf{z})\,[-igt\cdot\mathcal{F}_j^{\ -}(z'^+,\mathbf{z})]\,\mathcal{U}_F(z'^+,-\infty;\mathbf{z})$$

$$\mathcal{U}^{(3)}_{F;ij}(\mathbf{z}) = \int dz^+\,\mathcal{U}_F(+\infty,z^+;\mathbf{z})[gt\cdot\mathcal{F}_{ij}(z^+,\mathbf{z})]\,\mathcal{U}_F(z^+,-\infty;\mathbf{z})$$

Providing better understanding of the gluon content of the target beyond eikonal accuracy

NEik corrections in the CGC
quark background

An extra source of NEik corrections: **interaction with the quark background of the target**

The quark background field of the target can be split into good and bad components as

$$\Psi(z) = \Psi^{(-)}(z) + \Psi^{(+)}(z) \quad \text{with}$$

$$\Psi^{(-)}(z) \equiv \frac{\gamma^+\gamma^-}{2}\Psi(z), \quad \Psi^{(+)}(z) \equiv \frac{\gamma^-\gamma^+}{2}\Psi(z)$$

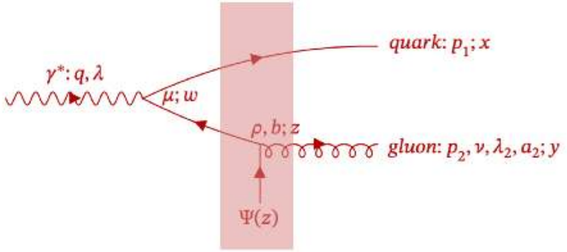
Under a boost of the target of parameter γ_t along the "-" direction, the projections scale as

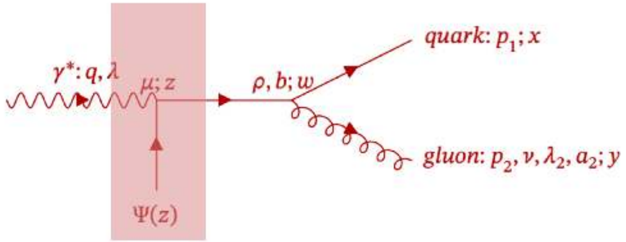
$$\Psi^{(-)}(z) \propto (\gamma_t)^{\frac{1}{2}}, \quad \Psi^{(+)}(z) \propto (\gamma_t)^{-\frac{1}{2}}$$

We keep only the leading component the quark background field

$$\Psi^{(-)}(z)$$

Suppressed compared to the enhanced component of the gluon background→quark background interaction only appears in NEik.
e.g., process quark-gluon dijet production in DIS





Probing the quark content of the target beyond eikonal accuacy

NEik CGC
state-of-the-art

TA, Beuf, et al. (2014-2026)

NEik corrections to quark and gluon propagators in a gluon background field with applications to (dense target)

- Forward parton-nucleus scattering at NEik accuracy (both quark and gluon production)
- Effects of dynamical background
- DIS dijet production in general kinematics
- DIS dijet production in the back-to-back limit (see talk from Guillaume Beuf)

TA, et al. (2016-2024)

NEik corrections to quark and gluon propagators in a gluon background field with applications to (dilute and dense target)

- Single and double inclusive parton production at NEik accuracy
- NEik corrections to Lipatov vertex in particle production
- NEik effects on azimuthal asymmetries and two gluon correlations in pp and pA collisions

TA, Beuf, et al. (2023-2026)

NEik corrections to observables in a quark background field with applications to (dense target)

- Quark-gluon dijet production in DIS
- Single parton production in forward parton-nucleus scattering with transition channels
- Dijet production in the back-to-back limit in pA collisions (see talk from Guillaume Beuf)

NEik CGC
state-of-the-art and road ahead

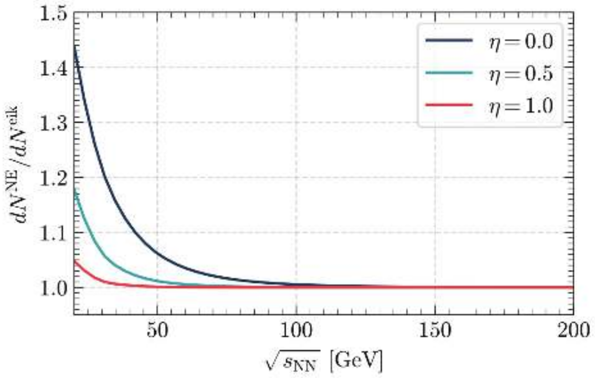
The numerical studies of NEik corrections - towards phenomenology

TA, et al.(2020-2024)

DISCLAIMER exploratory models/not all NEik effects are taken into account

- Dijet production in DIS off a dilute target (including only corrections to shockwave)
- Single and double inclusive gluon production in a dense target within harmonic oscillator approximation

CHALLENGE existing models can not explain the structures appearing at NEik order! A new model is needed



Ratio of single inclusive production cross section: **(Eik+NEik)/Eik**

for $p_\perp = 2$ GeV and $\sqrt{s_{NN}}$ ~40 GeV - 5% for $\eta = 0.5$

10 % for $\eta = 0.0$

Agostini, TA, Armesto, Beuf, Cougoulic, Mulani—2025

A Gaussian model to study the NEik color structures are recently developed

NEik dipole operator of type-1

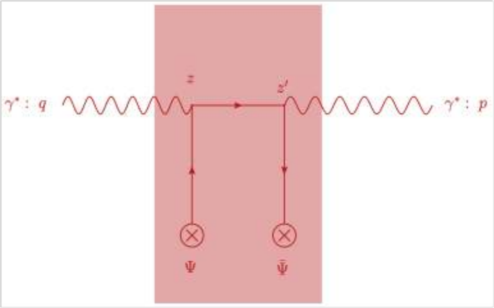
$$d_j^{(1)}(\mathbf{v}_*, \mathbf{w}) = \frac{1}{2}\frac{iQ_s^2}{P_q^-}(\mathbf{v}-\mathbf{w})^j \ln\left(\frac{1}{|\mathbf{v}-\mathbf{w}|\Lambda_{\mathrm{QCD}}}\right)\exp\left\{-\frac{Q_s^2}{4}(\mathbf{v}-\mathbf{w})^2 \ln\frac{1}{|\mathbf{v}-\mathbf{w}|\Lambda_{\mathrm{QCD}}}\right\}$$

We are working on the numerical implementation of our new model for DIS dijet production at NEik accuracy.

NEik CGC
state-of-the-art and road ahead
Loop corrections to the NEik observables

TA, Beuf, Mulani-2026

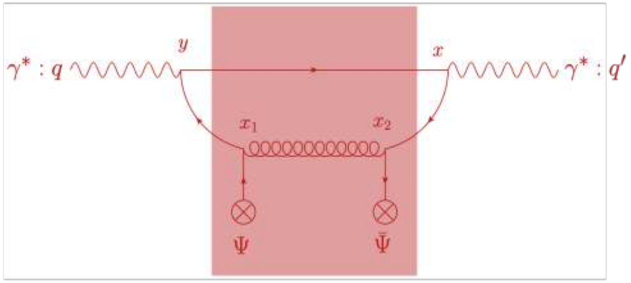
NEik quark background field contribution to the leading order (LO) DIS structure functions are computed.



$$F_T(x_{Bj},Q^2)\Big|_{q\text{ Backgd.}} = F_2(x_{Bj},Q^2)\Big|_{q\text{ Backgd.}} = 2x_{Bj}\,F_1(x_{Bj},Q^2)\Big|_{q\text{ Backgd.}}$$
$$= \sum_f e_f^2\, x_{Bj}\left[q_f(x_{Bj}) + \bar{q}_f(x_{Bj})\right] + \text{NNEik}$$
$$F_L(x_{Bj},Q^2)\Big|_{q\text{ Backgd.}} = 0 + \text{NNEik}\,,$$

TA, Beuf, Favrel, Fucilla-2026

NLO corrections to the NEik quark background field contribution to the DIS structure functions are also computed!



- NLO corrections to NEik longitudinal structure function are finite.
- NLO corrections to NEik transverse structure suffer from rapidity and UV divergences:
  - Finite NLO contributions are extracted.
  - Rapidity and UV divergences are analyzed.

We are working on the derivation of the evolution equations of the collinear operators.

Next step NLO corrections to NEik quark background contribution to the semi-inclusive deep inelastic scattering (SIDIS).

First study for loop corrections to the NEik observables

Thank you