

# Interplay between High energy QCD and TMD factorization beyond leading power

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# Motivation

Bound-state problem unsolved in QCD (nonperturbative, confinement, ...)

⇒ Collisions involving hadrons always contaminated by nonperturbative effects.

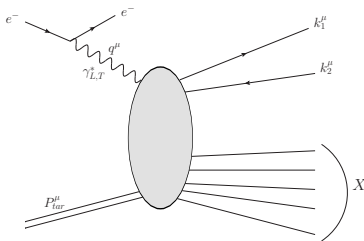
In specific kinematic ranges, factorization formalisms allow to address this issue:

- For hard processes, **collinear or TMD factorization** separates perturbatively calculable short distance dynamics, from nonperturbative but universal long range dynamics, parametrized by parton distributions.  
→ QCD-improved **parton model picture**
- For the high-energy limit of not-so-hard processes, **CGC formalism and dipole factorization**: separation between short time dynamics and long time dynamics  
→ Semi-classical picture of QCD based on **gluon background fields** for the hadrons  
Accounts for non-linear **gluon saturation** effects

⇒ Different formalisms for QCD scattering, based on very different pictures and degrees of freedom

In this talk: *Studies of the consistency between CGC and TMD approaches, at leading order in  $\alpha_s$  but beyond leading powers in energy or momentum*

# Dijet production in deep inelastic scattering (DIS)



Photon virtuality:  $Q^2 = -q_\mu q^\mu$

Photon-target center of mass energy:  
 $W^2 = (q + P_{tar})^2$

Measured jets:  $k_1^\mu$  and  $k_2^\mu$

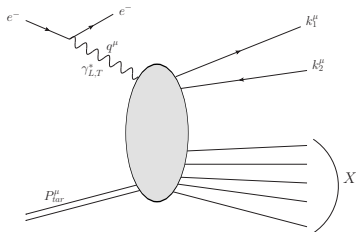
Conventions:

Light-cone variables:  $x^\pm = \frac{(x^0 \pm x^3)}{\sqrt{2}}$

Photon going in positive direction along the  $x^3$  axis, and proton or nucleus target along the negative direction

$\Rightarrow W^2 \sim 2q^+ P_{tar}^-$  at high collision energy

# TMD factorization for DIS dijet



Change of variables for the jets:

Jets light-cone momentum fractions:

$$z_1 = k_1^+ / (k_1^+ + k_2^+) \text{ and}$$

$$z_2 = k_2^+ / (k_1^+ + k_2^+) = 1 - z_1$$

Dijet imbalance:  $\mathbf{k} = \mathbf{k}_1 + \mathbf{k}_2$

Relative jet momentum:

$$\mathbf{P} = (z_2 \mathbf{k}_1 - z_1 \mathbf{k}_2)$$

Back-to-back (hard) jets production regime:  $|\mathbf{k}| \ll |\mathbf{P}| \sim W$

→ TMD factorization, valid up to power-suppressed corrections in  $|\mathbf{k}|/|\mathbf{P}|$ :

$$\left. \frac{d\sigma_{\gamma_{T,L}^* \rightarrow jj}}{dz_1 d^2\mathbf{P} d^2\mathbf{k}} \right|_{\text{TMD}} = \left[ \mathcal{H}_{T,L}^{f_1^g}(z_1, \mathbf{P}, Q^2) \times f_1^g(x, \mathbf{k}) + \mathcal{H}_{T,L}^{h_1^{\perp g}}(z_1, \mathbf{P}, Q^2) \times h_1^{\perp g}(x, \mathbf{k}) \right. \\ \left. + \mathcal{H}_{T,L}^{f_1^q}(z_1, \mathbf{P}, Q^2) \times f_1^q(x, \mathbf{k}) \right] \Big|_{x = \frac{\mathbf{P}^2 + m^2}{z_1 z_2 W^2} + \frac{Q^2}{W^2}} + \text{NLP}$$

- Unpolarized gluon TMD distribution  $f_1^g(x, \mathbf{k})$  and unpolarized quark TMD distribution  $f_1^q(x, \mathbf{k})$
- Linearly polarized gluon TMD distribution  $h_1^{\perp g}(x, \mathbf{k})$

→ Picks a parton in the target with momentum  $(xP_{tar}^-, \mathbf{k})$

(At higher order in  $\alpha_s$ , extra dependence on two factorization scales, from CSS resummation of large logs)

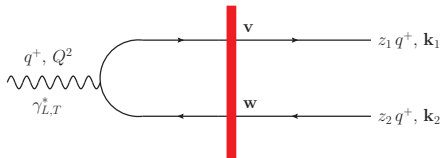
# Dipole factorization for DIS dijet

Other regime: Regge-Gribov high-energy limit:  $Q^2, \mathbf{k}_1^2, \mathbf{k}_2^2 \ll W^2 \rightarrow +\infty$

→ Dipole factorization, valid up to power-suppressed corrections in  $\mathbf{k}_{1,2}^2/W^2$

Leading power at high energy corresponds to Eikonal approximation (see talk from Tolga):

- Static gluon field  $\mathcal{A}^\mu(z) = \mathcal{A}^\mu(z^+, \mathbf{z}, 0)$  due to large Lorentz time dilation of the target
- Shockwave limit  $\mathcal{A}^\mu(z) \propto \delta(z^+)$  due to large Lorentz length contraction of the target
- Only  $\mathcal{A}^-$  component of gluon field is enhanced by large boost of the target



DIS dijet amplitude in the dipole factorization:

$$i\mathcal{M}_{q_1 \bar{q}_2 \leftarrow \gamma_{L,T}^*}^{\text{Eik}} \sim \int_{\mathbf{v}, \mathbf{w}} e^{-i\mathbf{v} \cdot \mathbf{k}_1} e^{-i\mathbf{w} \cdot \mathbf{k}_2} f_{L,T}(z_1, \mathbf{v} - \mathbf{w}, Q^2) \left[ \mathcal{U}_F(\mathbf{v}) \mathcal{U}_F^\dagger(\mathbf{w}) - 1 \right]$$

Coherent multiple scattering of quark and antiquark on the target resummed via Wilson line:

$$\mathcal{U}_F(\mathbf{z}) \equiv \mathbf{1} + \sum_{N=1}^{+\infty} \frac{1}{N!} \mathcal{P}_+ \left[ -ig \int dz^+ t \cdot \mathcal{A}^-(z^+, \mathbf{z}, 0) \right]^N$$

⇒ Non-linear gluon saturation effects included

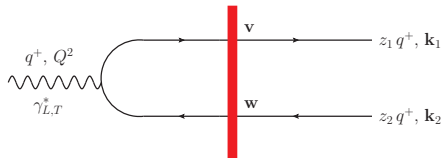
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$$\mathbf{k} = \mathbf{k}_1 + \mathbf{k}_2$$

$$\mathbf{P} = (z_2 \mathbf{k}_1 - z_1 \mathbf{k}_2)$$

DIS dijet amplitude in the dipole factorization:

$$i\mathcal{M}_{q_1 \bar{q}_2 \leftarrow \gamma_{L,T}^*}^{\text{Eik}} \sim \int_{\mathbf{r}} e^{-i\mathbf{r} \cdot \mathbf{P}} f_{L,T}(z_1, \mathbf{r}, Q^2) \int_{\mathbf{b}} e^{-i\mathbf{b} \cdot \mathbf{k}} \left[ \mathcal{U}_F(\mathbf{b} + z_2 \mathbf{r}) \mathcal{U}_F^\dagger(\mathbf{b} - z_1 \mathbf{r}) - 1 \right]$$

New variables:

- Dipole vector  $\mathbf{r} = \mathbf{v} - \mathbf{w}$ , conjugate to  $\mathbf{P}$
- Dipole impact parameter  $\mathbf{b} = z_1 \mathbf{v} + z_2 \mathbf{w}$ , conjugate to  $\mathbf{k}$

# Interplay between TMD and high-energy approaches

- TMD factorization: leading power (LP) in the back-to-back limit  $|\mathbf{k}| \ll |\mathbf{P}| \sim W$
- Dipole factorization: leading power (eikonal) in the high-energy limit  $|\mathbf{k}| \sim |\mathbf{P}| \ll W$

Consistency of both approaches known in the double limit  $|\mathbf{k}| \ll |\mathbf{P}| \ll \sqrt{s}$  at leading power (Dominguez, Marquet, Xiao, Yuan, 2011)

⇒ How are the two formalisms matching when including subleading power corrections?

- Next-to-leading power corrections (NLP) in the back-to-back jets regime, suppressed as  $|\mathbf{k}|/|\mathbf{P}|$
- Next-to-eikonal (NEik) corrections in the high-energy regime, suppressed as  $\mathbf{P}^2/W^2$
- Mixed NLP and NEik corrections, suppressed as  $|\mathbf{P}||\mathbf{k}|/W^2$

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Method:

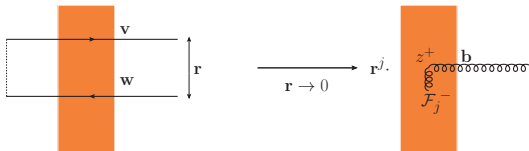
- 1 Calculate NEik corrections to the DIS dijet amplitude in the dipole factorization
  - Relaxing the static approximation for the target gluon field
  - Relaxing the shockwave approximation
  - Including the transverse components of the target gluon field  
(see talk from Tolga)
- 2 Expand the result in the back-to-back jets regime  $|\mathbf{k}| \ll |\mathbf{P}|$
- 3 Interpret the result in terms of TMD factorization

# Back-to-back regime: small dipole $\mathbf{r}$ expansion

Back-to-back expansion  $\mathbf{P} \gg \mathbf{k}$  of the Eikonal DIS dijet amplitude:

→ Equivalent to small dipole  $\mathbf{r} \ll \mathbf{b}$  expansion

$$\begin{aligned}
 & \int_{\mathbf{r}} e^{-i\mathbf{r}\cdot\mathbf{P}} f_{L,T}(z_1, \mathbf{r}, Q^2) \int_{\mathbf{b}} e^{-i\mathbf{b}\cdot\mathbf{k}} \left[ \mathcal{U}_F(\mathbf{b} + z_2 \mathbf{r}) \mathcal{U}_F^\dagger(\mathbf{b} - z_1 \mathbf{r}) - 1 \right] \\
 &= \int_{\mathbf{r}} e^{-i\mathbf{r}\cdot\mathbf{P}} f_{L,T}(z_1, \mathbf{r}, Q^2) \int_{\mathbf{b}} e^{-i\mathbf{b}\cdot\mathbf{k}} \left[ z_2 \mathbf{r}^j (\partial_j \mathcal{U}_F(\mathbf{b})) \mathcal{U}_F^\dagger(\mathbf{b}) - z_1 \mathbf{r}^j \mathcal{U}_F(\mathbf{b}) (\partial_j \mathcal{U}_F^\dagger(\mathbf{b})) + O(\mathbf{r}^2) \right] \\
 &= \int_{\mathbf{r}} e^{-i\mathbf{r}\cdot\mathbf{P}} f_{L,T}(z_1, \mathbf{r}, Q^2) \left[ \mathbf{r}^j t^a \int_{\mathbf{b}} e^{-i\mathbf{b}\cdot\mathbf{k}} \int_{z^+} \mathcal{U}_A(+\infty, z^+; \mathbf{b})_{ab} (-ig) \mathcal{F}_j^{b-}(z^+, \mathbf{b}) + O(\mathbf{r}^2) \right]
 \end{aligned}$$



Note: 0th order in the  $\mathbf{r}$  expansion trivial → first order in  $\mathbf{r}$  is the leading power

In our study (Altinoluk, G.B., Czajka, Marquet (2025)):

- Calculation of  $O(\mathbf{r}^2)$  terms: NLP in the back-to-back regime
- Small  $\mathbf{r}$  expansion also performed for the NEik corrections (→ LP and NLP terms)

# Back-to-back cross section: $\langle \mathcal{F}^{\perp-} \mathcal{F}^{\perp-} \rangle$ contributions

Including all contributions of the form  $\langle \mathcal{F}^{\perp-} \mathcal{F}^{\perp-} \rangle$ , of order **Eik** or **NEik**, and **LP** or **NLP** in the back-to-back regime:

$$\begin{aligned} \left. \frac{d\sigma_{\gamma_L^+ \rightarrow q_1 \bar{q}_2}}{dP.S.} \right|_{\mathcal{F}^{\perp-} \mathcal{F}^{\perp-}} &= (2q^+) 2\pi \delta(k_1^+ + k_2^+ - q^+) (ee_f)^2 g^2 4z_1^3 z_2^3 Q^2 \\ &\times \left[ \frac{4\mathbf{P}^i \mathbf{P}^j}{(\mathbf{P}^2 + \bar{Q}^2)^4} - 2(z_2 - z_1) \frac{(\mathbf{P}^i \mathbf{k}^j + \mathbf{k}^i \mathbf{P}^j)}{[\mathbf{P}^2 + \bar{Q}^2]^4} + 16(z_2 - z_1) \frac{(\mathbf{k} \cdot \mathbf{P}) \mathbf{P}^i \mathbf{P}^j}{[\mathbf{P}^2 + \bar{Q}^2]^5} + \mathcal{O}\left(\frac{\mathbf{k}^2}{\mathbf{P}^8}\right) \right] \\ &\times \int_{\mathbf{b}, \mathbf{b}'} e^{-i\mathbf{k} \cdot (\mathbf{b} - \mathbf{b}')} \int_{z^+, z'^+} \left[ 1 + i(z^+ - z'^+) \frac{(\mathbf{P}^2 + \bar{Q}^2)}{2q^+ z_1 z_2} + \text{NNEik} \right] \\ &\times \langle \mathcal{F}_a^{i-}(z'^+, \mathbf{b}') \left[ \mathcal{U}_A^\dagger(+\infty, z'^+; \mathbf{b}') \mathcal{U}_A(+\infty, z^+; \mathbf{b}) \right]_{ab} \mathcal{F}_b^{j-}(z^+, \mathbf{b}) \rangle \end{aligned}$$

With  $\bar{Q}^2 = m^2 + z_1 z_2 Q^2$

- **NEik corrections** and **NLP corrections** in the  $\langle \mathcal{F}^{\perp-} \mathcal{F}^{\perp-} \rangle$  contribution factorize from each other

Notation: TMD  $\langle \mathcal{F}^{i-} \mathcal{F}^{j-} \rangle$  correlator in unpolarized target (with the target mass  $M$ ):

$$\begin{aligned} x\Phi^{i-j-}(x, \mathbf{k}) &= \frac{2}{(2\pi)^3} \int_{\mathbf{b}, \mathbf{b}'} e^{-i\mathbf{k} \cdot (\mathbf{b} - \mathbf{b}')} \int_{z^+, z'^+} e^{ixP_{tar}^-(z^+ - z'^+)} \langle \mathcal{F}_a^{i-}(z'^+, \mathbf{b}') \left[ \mathcal{U}_A^\dagger(+\infty, z'^+; \mathbf{b}') \mathcal{U}_A(+\infty, z^+; \mathbf{b}) \right]_{ab} \mathcal{F}_b^{j-}(z^+, \mathbf{b}) \rangle \\ &= \frac{\delta^{ij}}{2} x f_1^g(x, \mathbf{k}) + \left[ \mathbf{k}^i \mathbf{k}^j - \frac{\mathbf{k}^2}{2} \delta^{ij} \right] \frac{1}{2M^2} x h_1^{\perp g}(x, \mathbf{k}) \end{aligned}$$

- Missing phase in Eikonal contribution  $\Rightarrow$  Recovering gluon TMDs but at  $x = 0$  from Eikonal result
- NEik correction can be interpreted as start of the expansion of the phase
  - $\Rightarrow$   $x$  dependence of TMDs recovered from an all order resummation of power corrections to eikonal approximation
  - $\Rightarrow$  DIS dijets production probes TMDs at  $x = \frac{[\mathbf{P}^2 + \bar{Q}^2]}{z_1 z_2 W^2}$ : ok!

# Back-to-back DIS dijets: from target gluon field

$$\frac{d\sigma_{\gamma_{T,L}^* \rightarrow q_1 \bar{q}_2}}{dz_1 d^2\mathbf{P} d^2\mathbf{k}} \Big|_{\text{Eik+NEik}} = \alpha_{\text{em}} e_f^2 \alpha_s \left\{ C_{T,L}^{f_1^g}(z_1, \mathbf{P}, \mathbf{k}) \times f_1^g(x, \mathbf{k}) + C_{T,L}^{h_1^{\perp g}}(z_1, \mathbf{P}, \mathbf{k}) \times h_1^{\perp g}(x, \mathbf{k}) \right. \\ \left. + \frac{(\mathbf{k} \cdot \mathbf{P})}{W^2} C_{T,L}^{f^{\perp g}}(z_1, \mathbf{P}) \times f^{\perp g}(x, \mathbf{k}) + \frac{(\mathbf{k} \cdot \mathbf{P})}{W^2} C_{T,L}^{\bar{g}^{\perp g}}(z_1, \mathbf{P}) \times \bar{g}^{\perp g}(x, \mathbf{k}) \right\} \Big|_{x = \frac{|\mathbf{P}^2 + \bar{Q}^2|}{z_1 z_2 W^2}} \\ + 3 \mathcal{F} \text{ terms} + O\left(\frac{1}{\mathbf{P}^6}\right)$$

Altinoluk, G.B., Czajka, Marquet (2025)

Extra terms: all power suppressed in the back-to-back jets regime (NLP)

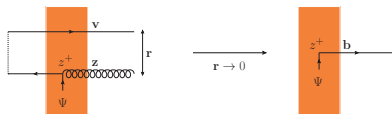
- NLP gluon TMD  $f^{\perp g}(x, \mathbf{k})$ , defined from  $\langle \mathcal{F}^{+-} \mathcal{F}^{\perp -} \rangle$   
→ NEik correction beyond the static approximation for the target fields
- NLP gluon TMD  $\bar{g}^{\perp g}(x, \mathbf{k})$ , defined from  $\langle \mathcal{F}^{12} \mathcal{F}^{\perp -} \rangle$   
→ NEik correction due to the transverse components of the target field
- Extra NLP corrections beyond TMD parton distributions, of the type  $\langle \mathcal{F}^{\perp -} \mathcal{F}^{\perp -} \mathcal{F}^{\perp -} \rangle$   
→ Contain both Eik and NEik contributions. Genuine saturation corrections: nonlinear QCD.  
Dedicated study of these contributions to appear soon: Altinoluk, G.B., Czajka, Gosławski

# Back-to-back DIS dijets: from target quark field

Another NEik contribution to dijet production in DIS:  
quark-gluon dijet from quark background field of the target.



At LP in the back-to-back limit  $\mathbf{P} \gg \mathbf{k}$  (or small dipole  $\mathbf{r} \ll \mathbf{b}$  expansion):



$$\frac{d\sigma_{\gamma_{T,L}^* \rightarrow q_1 g_2}}{dz_1 d^2\mathbf{P} d^2\mathbf{k}} \Big|_{\text{TMD}} = \frac{1}{W^2} \mathcal{H}_{T,L}^{f_1^q}(z_1, \mathbf{P}, Q^2) f_1^q(x=0, \mathbf{k})$$

Contribution of quark TMD from the target is recovered, at  $x = 0$

Altinoluk, Armesto, G.B. (2023)

Further ongoing study: calculation of NLP corrections, including three fields correlators of the type  $\langle \bar{\Psi} \mathcal{F}^{\perp-} \Psi \rangle$

Altinoluk, G.B. Czajka

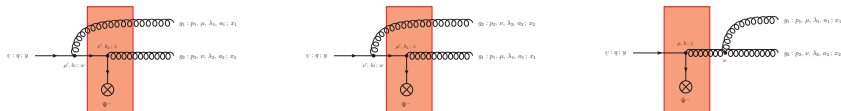
# Back-to-back dijets in pA: from target quark field

Similar study but in proton-nucleus collisions: [Altinoluk, G.B., Blanco, Mulani \(2025\)](#)

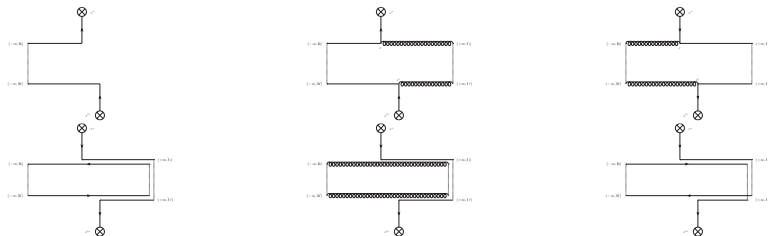
Back-to-back dijet production at LP, from the quark background field of the target nucleus (NEik)

Multiple channels, induced by quark, antiquark or gluon from the proton:  $g \rightarrow gq$ ,  $q \rightarrow q\bar{q}$ ,  $q \rightarrow gg$ ,  $q \rightarrow qq$  and symmetric ones by  $q \leftrightarrow \bar{q}$

Ex:  $q \rightarrow gg$  channel:



All channels obtained as linear combinations of quark TMDs at  $x = 0$ , but with different gauge link structures:



# Summary

To understand the interplay between high-energy (CGC) and TMD formalisms for QCD, we studied the dijet production in the back-to-back jets limit, at subleading power in energy (NEik)

TMD formalism properly recovered from NEik CGC calculations

- gluon TMDs appear at Eik order, but with  $x = 0$
- Nontrivial  $x$  dependence recovered by resumming NEik corrections
- quark TMDs appear at NEik order (with  $x = 0$ )

NLP corrections in  $|\mathbf{k}|/|\mathbf{P}|$  include

- Kinematical corrections to the hard coefficients
- Contributions from subleading TMDs, such as 3 fields correlators

Gauge links for the TMDs:

- Always staple shape oriented towards the future for DIS dijets
- Multiple exotic shapes for the gauge link occurs for dijet production in pA

# Final result: $\langle \mathcal{F}\mathcal{F} \rangle$ contributions to the cross sections

$$\begin{aligned} \left. \frac{d\sigma_{\gamma_{T,L}^* \rightarrow q_1 \bar{q}_2}}{dz_1 d^2\mathbf{P} d^2\mathbf{k}} \right|_{\text{Eik}+\text{NEik}} &= \alpha_{\text{em}} e_f^2 \alpha_s \left\{ \mathcal{C}_{T,L}^{f_1^g}(z_1, \mathbf{P}, \mathbf{k}) \times f_1^g(x, \mathbf{k}) + \mathcal{C}_{T,L}^{h_1^{\perp g}}(z_1, \mathbf{P}, \mathbf{k}) \times h_1^{\perp g}(x, \mathbf{k}) \right. \\ &\quad \left. + \frac{(\mathbf{k} \cdot \mathbf{P})}{W^2} \mathcal{C}_{T,L}^{f^{\perp g}}(z_1, \mathbf{P}) \times f^{\perp g}(x, \mathbf{k}) + \frac{(\mathbf{k} \cdot \mathbf{P})}{W^2} \mathcal{C}_{T,L}^{\bar{g}^{\perp g}}(z_1, \mathbf{P}) \times \bar{g}^{\perp g}(x, \mathbf{k}) \right\} \Bigg|_{x=\frac{[\mathbf{P}^2+\bar{Q}^2]}{z_1 z_2 W^2}} \\ &\quad + 3\mathcal{F} \text{ terms} + O\left(\frac{1}{\mathbf{P}^6}\right) \end{aligned}$$

Contribution from longitudinal photon exchange:

$$\begin{aligned} \mathcal{C}_L^{f_1^g}(z_1, \mathbf{P}, \mathbf{k}) &= \frac{8Q^2 z_1^2 z_2^2}{[\mathbf{P}^2 + \bar{Q}^2]^4} \left\{ \mathbf{P}^2 + (z_2 - z_1)(\mathbf{k} \cdot \mathbf{P}) \left[ -1 + \frac{4\mathbf{P}^2}{[\mathbf{P}^2 + \bar{Q}^2]} \right] \right\} + O\left(\frac{Q^2 \mathbf{k}^2}{\mathbf{P}^8}\right) \\ \mathcal{C}_L^{h_1^{\perp g}}(z_1, \mathbf{P}, \mathbf{k}) &= \frac{4Q^2 z_1^2 z_2^2}{[\mathbf{P}^2 + \bar{Q}^2]^4} \frac{\mathbf{k}^2}{M^2} \left\{ \left( \frac{2(\mathbf{k} \cdot \mathbf{P})^2}{\mathbf{k}^2 \mathbf{P}^2} - 1 \right) \mathbf{P}^2 \right. \\ &\quad \left. + (z_2 - z_1)(\mathbf{k} \cdot \mathbf{P}) \left[ -1 + \frac{4\mathbf{P}^2}{[\mathbf{P}^2 + \bar{Q}^2]} \left( \frac{2(\mathbf{k} \cdot \mathbf{P})^2}{\mathbf{k}^2 \mathbf{P}^2} - 1 \right) \right] \right\} + O\left(\frac{Q^2 \mathbf{k}^2}{\mathbf{P}^8}\right) \\ \mathcal{C}_L^{f_1^{\perp g}}(z_1, \mathbf{P}) &= -32Q^2 z_1 z_2 \frac{(z_2 - z_1)[\mathbf{P}^2 + m^2]}{[\mathbf{P}^2 + \bar{Q}^2]^4} + O\left(\frac{Q^2 |\mathbf{k}|}{|\mathbf{P}|^7}\right) \\ \mathcal{C}_L^{\bar{g}_1^{\perp g}}(z_1, \mathbf{P}) &= 0 \end{aligned}$$

$$\bar{Q}^2 = m^2 + z_1 z_2 Q^2.$$

# Final result: $\langle \mathcal{F} \mathcal{F} \rangle$ contributions to the cross sections

$$\begin{aligned} \frac{d\sigma_{\gamma_{T,L}^* \rightarrow q_1 \bar{q}_2}}{dz_1 d^2\mathbf{P} d^2\mathbf{k}} \Big|_{\text{Eik+NEik}} &= \alpha_{\text{em}} e_f^2 \alpha_s \left\{ C_{T,L}^{f_1^g}(z_1, \mathbf{P}, \mathbf{k}) \times f_1^g(x, \mathbf{k}) + C_{T,L}^{h_1^{\perp g}}(z_1, \mathbf{P}, \mathbf{k}) \times h_1^{\perp g}(x, \mathbf{k}) \right. \\ &\quad \left. + \frac{(\mathbf{k} \cdot \mathbf{P})}{W^2} C_{T,L}^{f_1^{\perp g}}(z_1, \mathbf{P}) \times f_1^{\perp g}(x, \mathbf{k}) + \frac{(\mathbf{k} \cdot \mathbf{P})}{W^2} C_{T,L}^{\bar{g}^{\perp g}}(z_1, \mathbf{P}) \times \bar{g}^{\perp g}(x, \mathbf{k}) \right\} \Big|_{x=\frac{[\mathbf{P}^2+Q^2]}{z_1 z_2 W^2}} \\ &\quad + 3\mathcal{F} \text{ terms} + O\left(\frac{1}{\mathbf{P}^6}\right) \end{aligned}$$

Contribution from transverse photon exchange:

$$\begin{aligned} C_T^{f_1^g}(z_1, \mathbf{P}, \mathbf{k}) &= -\frac{2[(z_1^2+z_2^2)\bar{Q}^2-m^2]}{[\mathbf{P}^2+Q^2]^4} \left\{ \mathbf{P}^2 + (z_2-z_1)(\mathbf{k} \cdot \mathbf{P}) \left[ -1 + \frac{4\mathbf{P}^2}{[\mathbf{P}^2+Q^2]} \right] \right\} \\ &\quad + \frac{(z_1^2+z_2^2)}{[\mathbf{P}^2+Q^2]^2} \left[ 1 + \frac{2(z_2-z_1)(\mathbf{k} \cdot \mathbf{P})}{[\mathbf{P}^2+Q^2]} \right] + O\left(\frac{\mathbf{k}^2}{\mathbf{P}^6}\right) \\ C_T^{h_1^{\perp g}}(z_1, \mathbf{P}, \mathbf{k}) &= -\frac{[(z_1^2+z_2^2)\bar{Q}^2-m^2]}{[\mathbf{P}^2+Q^2]^4} \frac{\mathbf{k}^2}{M^2} \left\{ \left( \frac{2(\mathbf{k} \cdot \mathbf{P})^2}{\mathbf{k}^2 \mathbf{P}^2} - 1 \right) \mathbf{P}^2 \right. \\ &\quad \left. + (z_2-z_1)(\mathbf{k} \cdot \mathbf{P}) \left[ -1 + \frac{4\mathbf{P}^2}{[\mathbf{P}^2+Q^2]} \left( \frac{2(\mathbf{k} \cdot \mathbf{P})^2}{\mathbf{k}^2 \mathbf{P}^2} - 1 \right) \right] \right\} + O\left(\frac{\mathbf{k}^2}{\mathbf{P}^6}\right) \\ C_T^{f_1^{\perp g}}(z_1, \mathbf{P}) &= \frac{4(z_2-z_1)}{[\mathbf{P}^2+Q^2]^4} \left\{ [\mathbf{P}^2+\bar{Q}^2 + (z_1^2+z_2^2)Q^2][\mathbf{P}^2-\bar{Q}^2] + 2m^2Q^2 \right\} + O\left(\frac{|\mathbf{k}|}{|\mathbf{P}|^5}\right) \\ C_T^{\bar{g}^{\perp g}}(z_1, \mathbf{P}) &= \frac{4(z_2-z_1)}{[\mathbf{P}^2+Q^2]^2} + O\left(\frac{|\mathbf{k}|}{|\mathbf{P}|^5}\right) \\ \bar{Q}^2 &= m^2 + z_1 z_2 Q^2. \end{aligned}$$